ZAINEMATICS

FOR THE LOVE OF MATHS

A - Level

P1 Notes

By

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Compiled By Rafay Mushtaq

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Coordinate Geometry P1

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COORDINATE GEOMETRY

Monday, 6 July 2020 5:15 PM

Chapter 1. Coordinate Geometry (7+4 marks)

merged with other topics.

$\boxed{15\% \text{ of P1.}}$

$A(x_1, y_1) \quad B(x_2, y_2)$

MIDPOINT $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

DISTANCE $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

GRADIENT (SLOPE) (m) $= \frac{y_2 - y_1}{x_2 - x_1}$

$A(1, 3) \quad B(5, 12)$

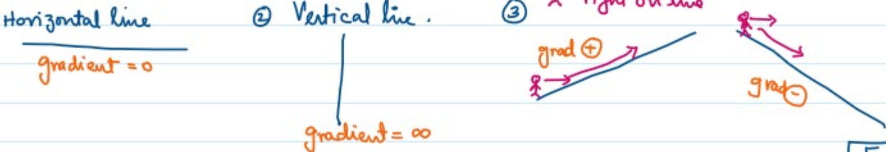
Midpoint $= \left(\frac{1+5}{2}, \frac{3+12}{2}\right)$
$= (3, 7.5)$

Distance $= \sqrt{(5-1)^2 + (12-3)^2}$
$= \sqrt{4^2 + 9^2}$
$= \sqrt{97}$

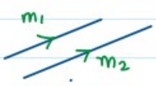
Gradient $= \frac{12-3}{5-1} = \frac{9}{4}$

GRADIENTS: (STEEPNESS)

① Horizontal line ② Vertical line. ③ will move left to right on line



PARALLEL LINES: have same gradient.



$\boxed{m_1 = m_2}$

PERPENDICULAR LINES (90°)

$\boxed{m_1 m_2 = -1}$

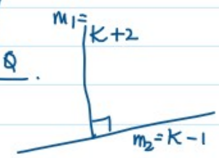
Shortcut.
Negative Reciprocal.

$m_1 = 3 \longrightarrow m_2 = -\frac{1}{3}$

$m_1 = -\frac{1}{5} \longrightarrow m_2 = +5$

$m_1 = \frac{3}{4} \longrightarrow m_2 = -\frac{4}{3}$

Exam Q.



Find K.

$m_1 \times m_2 = -1$
$(K+2)(K-1) = -1$
Quatratic Solve.
⋮
$K = \square$

EQUATION OF A LINE

SLOPE-INTERCEPT FORM

$y = mx + c$

POINT-SLOPE FORM (MOST IMPORTANT).

$y - y_1 = m(x - x_1)$

TWO POINT FORM. OPTIONAL

$\frac{y - y_1}{\ } = \frac{x - x_1}{\ }$

SLOPE-INTERCEPT FORM

$$y = mx + c$$

Given: ① Slope (m)
② y-intercept (c)

POINT-SLOPE FORM (MOST IMPORTANT).

$$y - y_1 = m(x - x_1)$$

Given:-
① Slope (m)
② Point on line (x_1, y_1).

TWO POINT FORM.

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Given: Two points on line.
(x_1, y_1) (x_2, y_2).

Q Find equation of line that has gradient 3 and it cuts y axis at 1

$$y = mx + c$$

$$\boxed{y = 3x + 1}$$

Never used in A Levels

Q Find equation of line with slope 2 and it passes through (1, 5).

$m = 2$, $(\underset{x_1}{1}, \underset{y_1}{5})$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 2(x - 1)$$

$$y - 5 = 2x - 2$$

$$\boxed{y = 2x + 3}$$

Q Find equation of line that passes through $(\underset{x_1}{1}, \underset{y_1}{3})$ and $(\underset{x_2}{7}, \underset{y_2}{12})$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{12 - 3} = \frac{x - 1}{7 - 1}$$

$$\frac{y - 3}{\cancel{9}\,3} = \frac{x - 1}{\cancel{6}\,2}$$

$$2(y - 3) = 3(x - 1)$$

$$2y - 6 = 3x - 3$$

$$2y = 3x + 3$$

$$\boxed{y = \frac{3}{2}x + \frac{3}{2}}$$

Q Find equation of line with gradient 4 and it passes through (2, 7)

$m = 4$, $(\underset{x_1}{2}, \underset{y_1}{7})$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 4(x - 2)$$

$$y - 7 = 4x - 8$$

$$y = 4x - 8 + 7$$

$$\boxed{y = 4x - 1}$$

Q Find equation of line that passes throug (-2, 5) and has slope 5.

$m = 5$, $(\underset{x_1}{-2}, \underset{y_1}{5})$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 5(x - (-2))$$

$$y - 5 = 5(x + 2)$$

$$y = 5x + 10 + 5$$

$$\boxed{y = 5x + 15}$$

A LEVELS IS ABOUT DETAILED CLEAR WORKINGS

SIMULTANOUS EQUATION :-

O Levels.

(1) $2x + y = 7$

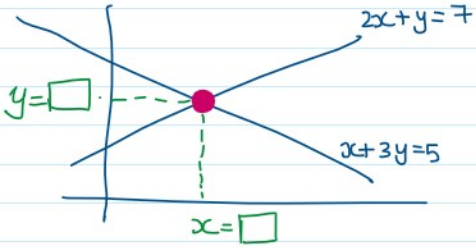
CONCEPT :-

SIMULTANEOUS EQUATION:-

$$2x + y = 7$$
$$x + 3y = 5$$

OLevels

$x = \square$, $y = \square$



SIMULTANOUS SOLVING WILL ALWAYS GIVE YOU POINTS OF INTERSECTION OF TWO GRAPHS.

BEST WAY TO SOLVE FOR POINT OF INTERSECTION. (SIMULTANOUS SOLVING).

1) Make y subject for both equations and put them equal.

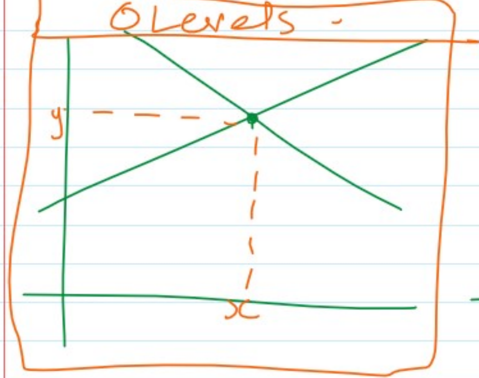
2) SUBTITUTION WILL ALWAYS WORK.

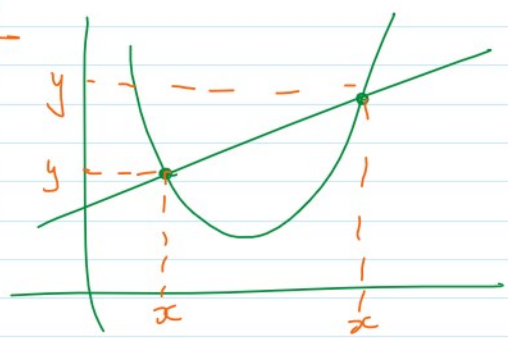
3) ELIMINATION WILL NOT ALWAYS BE AVAILABLE. (Specially when x or y has a power).

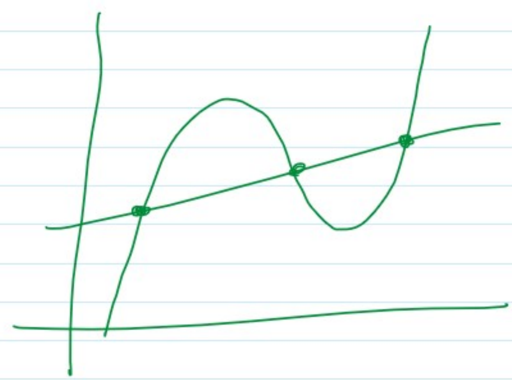
HOW MANY POINTS OF INTERSECTION DO YOU EXPECT IF TWO GRAPHS MEET

IT DEPENDS!

OLevels.







If power of x and y is 1
LINEAR GRAPH

If either x or y or both have any other power than 1,
NON LINEAR GRAPH
CURVE

LINEAR GRAPH

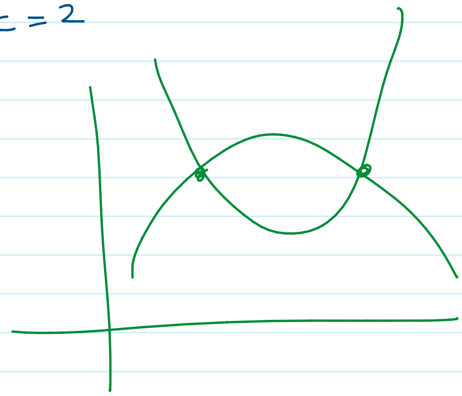
STRAIGHT LINE

$y = 2x + 5$

$2x - 3y + 7 = 0$

$y = 7$

$x = 2$



NON LINEAR GRAPH

CURVE

$y = x^2 + 2x$

$y = 3x^3$

$y = \frac{1}{x}$ $\quad$ $y = x^{-1}$

Curve.

$\sqrt{y} = 2x + 4$

1) Class Notes.

2) Past papes (Topical). I will upload these sheet.

Class → ~~Book~~

Class → Paper

HOW TO FIND EQUATION OF PERPENDICULAR BISECTOR OF TWO POINTS (4 marks)

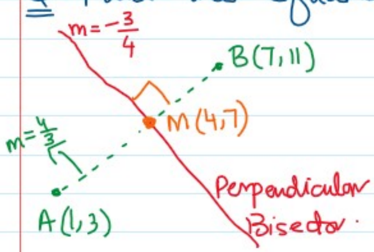
$90°$

Equal two parts.

HOW TO FIND EQUATION OF PERPENDICULAR BISECTOR OF TWO POINTS (4 marks)

Bisector: Equal two parts.

Q Find the equation of perpendicular bisector of A(1, 3) and B(7, 11).



STEP 1

Find coordinates of midpoint M.

$$M = \left(\frac{1+7}{2}, \frac{3+11}{2}\right)$$

$$M = (4, 7)$$

STEP 2

gradient of AB.

$$m_{AB} = \frac{11-3}{7-1} = \frac{8}{6} = \frac{4}{3}$$

Negative reciprocal.

$$m_{\perp} = -\frac{3}{4}$$

STEP 3: x_1, y_1

$m = -\frac{3}{4}$, M $(4, 7)$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = -\frac{3}{4}(x - 4)$$

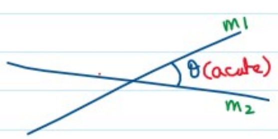
$$4y - 28 = -3x + 12$$

$$\boxed{4y = -3x + 40}$$

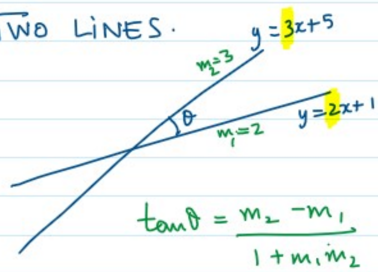
Symbol = $\perp$ means perpendicular

Note: No need to make y subject in A Levels.

ANGLE BETWEEN TWO LINES (ACUTE).



$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$



$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\tan\theta = \frac{3-2}{1+(3)(2)}$$

$$\tan\theta = \frac{1}{7}$$

$$\theta = \tan^{-1}\left(\frac{1}{7}\right)$$

$$\boxed{\theta = 8.13^\circ}$$

$m_1 = 3$, $m_2 = 2$

$$\tan\theta = \frac{2-3}{1+2(3)}$$

$$\tan\theta = \frac{-1}{7}$$

negative ignore

$$\theta = \tan^{-1}\left(\frac{1}{7}\right)$$

$$\theta = 8.13$$

[While taking inverse ignore any −ve sign. Reason: Discuss later in trigonometry.]

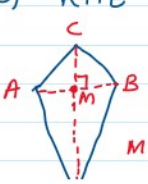
SPECIAL SHAPES

DIAGONALS MEET AT MIDPOINT

1) SQUARE
2) RHOMBUS
3) RECTANGLE.
4) PARALLELOGRAM.

DIAGONALS MEET AT 90°

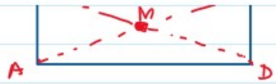
1) SQUARE
2) RHOMBUS.
3) KITE



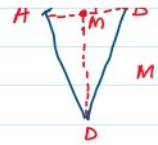
M is midpoint of AB

BUT

M is not midpoint of CD.



M is midpoint of AC and BD.



BUT
M is not midpoint of CD.

Q Find coordinates of D (1 marks).

Don't take (x, y) since there is a chance of mix up.

Midpoint of AC = Midpoint of BD.

$$\left(\frac{1+11}{2}, \frac{3+8}{2}\right) = \left(\frac{a+5}{2}, \frac{b+7}{2}\right)$$

$\frac{11+1}{\cancel{2}} = \frac{a+5}{\cancel{2}}$	$\frac{3+8}{\cancel{2}} = \frac{b+7}{\cancel{2}}$
$12 = a+5$	$11 = b+7$
$a = 7$	$b = 4$

$D \cdot (7, 4)$

HOW TO FIND GRADIENT OF A LINE FROM ITS EQUATION.

STEP 1 = Make y subject.
STEP 2 = Now compare with $y = mx + c$

$2y + 3x = 7$

$2y = -3x + 7$

$y = -\frac{3}{2}x + \frac{7}{2}$

$y = mx + c$

$\cancel{3}$, $\cancel{-3}$, $\cancel{\frac{3}{2}}$, $\frac{-3}{2}$

$\cancel{1.5}$, -1.5

IF A POINT LIES ON A LINE, IT MUST SATISFY ITS EQUATION

(O-levels)
Q Check if $(2, 3)$ lies on line $y = 2x + 1$ or not?

$y = 2x + 1$ $(2, 3)$

A levels.
Q Given that $(3, k)$ lies on $2x + 3y = 12$, find k. (1 or 2 marks).

$(3, k)$
$x \; y$

$2x + 3y = 12$

$2(3) + 3(k) = 12$

or not?

$y = 2x + 1 \qquad (2, 3)$

$3 = 2(2) + 1$

$3 \neq 5$

No, (2, 3) does not lie on $y = 2x + 1$.

$(3, k)$

$2x + 3y = 12$

$2(3) + 3(k) = 12$

$6 + 3k = 12$

$3k = 6$

$\boxed{k = 2}$

IF A LINE CUTS X-AXIS, put $y = 0$

IF A LINE CUTS Y-AXIS, put $x = 0$.

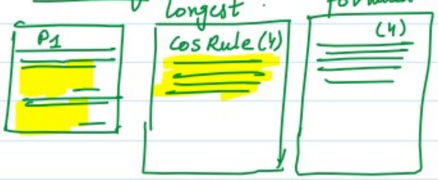
O-Levels v/s A levels

Length of chapter ↓

Length of Chapter ↑

STAMINA ↑

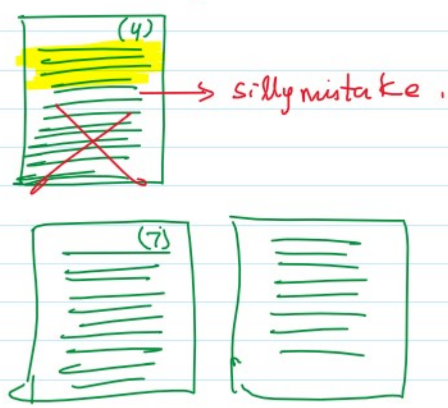
Workings



Workings.

P1 & M1.

- Normally go upto 1 page for 4-5 marks.
- Go upto 2 pages for 6-8 marks



ZAINEMATICS

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Binomial

P1

Compiled by Rafay Mushtaq

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BINOMIAL

Tuesday, 18 August 2020 5:06 PM

Coordinate Geometry Q1 — Q10 (Wed) (19 Aug 2020)

BINOMIAL (4–6 Marks).

$\boxed{\text{Total terms in a binomial} = n+1}$

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \ldots$$

$\boxed{{}^nC_r}$ → Button beneath Alpha $\boxed{nCr}$

→ $\boxed{\text{SHIFT}}\ \boxed{\div}$

Meaning will come in S1

${}^6C_3 = 6\ \boxed{nCr}\ 3$

$6C3 = 20$

Expand first three terms in following.

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2$$

(i) $(2+3x)^5 = 2^5 + {}^5C_1 (2)^{5-1}(3x)^1 + {}^5C_2 (2)^{5-2}(3x)^2$

Calculator = $\boxed{5C1 \times 2\text{^}4 \times 3}$ $\boxed{5C2 \times 2^3 \times 3^2}$

$\boxed{32 + 240x + 720x^2}$

(ii) $(5-4x)^4 = (5)^4 + {}^4C_1 (5)^{4-1}(-4x)^1 + {}^4C_2 (5)^{4-2}(-4x)^2$

$\boxed{4C1 \times 5^3 \times (-4)}$ $\boxed{4C2 \times 5^2 \times (-4)^2}$

$625 - 2000x + 2400x^2$

(iii) $(3+5x)^3$

$(3)^3 + {}^3C_1 (3)^{3-1}(5x)^1 + {}^3C_2 (3)^{3-2}(5x)^2$

$\boxed{3C2 \times 3^1 \times 5^2}$

$27 + 135x + 225x^2$

(iv) $\left(x + \frac{1}{x}\right)^7$

$(x)^7 + {}^7C_1 (x)^{7-1}\left(\frac{1}{x}\right)^1 + {}^7C_2 (x)^{7-2}\left(\frac{1}{x}\right)^2$

$x^7 + (7)\,\cancel{x^6}^{\,5}\left(\frac{1}{\cancel{x}}\right) + (21)(\cancel{x^5}^{\,3})\left(\frac{1}{\cancel{x^2}}\right)$

$\boxed{x^7 + 7x^5 + 21x^3}$

(v) Expand first three terms in expansion

$\left(3x - \frac{2}{x}\right)^5$

$(3x)^5 + {}^5C_1 (3x)^{5-1}\left(\frac{-2}{x}\right)^1 + {}^5C_2 (3x)^{5-2}\left(\frac{-2}{x}\right)^2$

$243x^5 + (5)(81\cancel{x^4}^{\,3})\left(\frac{-2}{\cancel{x}}\right) + (10)(27\cancel{x^3}^{\,1})\left(\frac{4}{\cancel{x^2}}\right)$

$243x^5 - 810x^3 + 1080x$

TYPE 1 QUESTION

(a) Expand $(2-x)^6$ upto first three terms.

$(a+b)^n$ = Total term $= n+1$

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots$$

$$(2-x)^6 = (2)^6 + {}^6C_1 (2)^{6-1}(-x)^1 + {}^6C_2 (2)^{6-2}(-x)^2$$

$$= \boxed{64 - 192x + 240x^2}$$

Hence
(ii) Find cofficient of x^2 in expansion

2 terms   7 terms

$(3x+5)(2-x)^6 \longrightarrow x^2$

$3x \times -192x = -576x^2$

$+5 \times 240x^2 = 1200x^2$

$\boxed{624x^2}$

$(2-x)^6 = \boxed{64 - 192x + 240x^2}$

(iii) Find cofficient of x^2 in expansion of

Terms = 3   Terms = 7

$(3 - 7x + 2x^2)(2-x)^6$

$3 \times 240x^2 = 720x^2$

$-7x \times -192x = 1344x^2$

$+2x^2 \times 64 = 128x^2$

$2192x^2$

(iii) Hence Find cofficient of x^3 in expansion of

2 terms   7 terms

$(2x + x^2)(2-x)^6 \longrightarrow x^3$

$2x \times 240x^2 = 480x^3$

$+x^2 \times -192x = -192x^3$

$\boxed{288x^3}$

(iv) Find cofficient of x in expansion of

$(7x+3)(2-x)^6 \longrightarrow x$

$7x \times 64 = 448x$

$+3 \times -192x = -576x$

$\boxed{-128x}$

25 (i) Find the first 3 terms in the expansion of $(2x - x^2)^6$ in ascending powers of x. [3]

(ii) Hence find the coefficient of x^8 in the expansion of $(2 + x)(2x - x^2)^6$. [2]

(i) $(2x - x^2)^6 = (2x)^6 + {}^6C_1 (2x)^5(-x^2)^1 + {}^6C_2 (2x)^4(-x^2)^2$

$= 64x^6 + (6)(32x^5)(-x^2) + (15)(16x^4)(x^4)$

$= 64x^6 - 192x^7 + 240x^8$

$(2+x)(2x-x^2)^6 \longrightarrow x^8$

(ii)

→ Student think

(ii) $(2+x)(2x-x^2)^6 \longrightarrow x^8$

$2 \times 240x^8 = 480x^8$
$+x \times -192x^7 = -192x^8$
$= 288x^8$

M1 → Student think 1 steps
→ In Real M1 = Complete Method.

VARIATION: Reverse WORKING.

9 (i) Find the first 3 terms in the expansion of $(2-x)^6$ in ascending powers of x. [3]

(ii) Given that the coefficient of x^2 in the expansion of $(1+2x+ax^2)(2-x)^6$ is 48, find the value of the constant a. [3]

3 terms 7 term

(i) $(2-x)^6 = (2)^6 + {}^6C_1(2)^5(-x)^1 + {}^6C_2(2)^4(-x)^2$
$= 64 - 192x + 240x^2$

(ii) $(1+2x+ax^2)(2-x)^6 \longrightarrow 48x^2$

$1 \times 240x^2 = 240x^2$
$+2x \times -192x = -384x^2$
$+ax^2 \times 64 = 64a\,x^2$
$= 48x^2$

$240 - 384 + 64a = 48$
$64a = 48 + 384 - 240$
$64a = 192$
$\boxed{a = 3}$

31 (i) Find the first three terms when $(2+3x)^6$ is expanded in ascending powers of x. [3]

(ii) In the expansion of $(1+ax)(2+3x)^6$, the coefficient of x^2 is zero. Find the value of a. [2]

(i) $(2+3x)^6 = (2)^6 + {}^6C_1(2)^5(3x)^1 + {}^6C_2(2)^4(3x)^2$
$= \boxed{64 + 576x + 2160x^2}$

(ii) $(1+ax)(2+3x)^6 \longrightarrow 0x^2$

$1 \times 2160x^2 = 2160x^2$
$+ax \times 576x = 576a\,x^2$
$= 0x^2$

$2160 + 576a = 0$
$576a = -2160$
$\boxed{a = -3.75}$

NEW VOCAB

Cofficient of x^2 is zero = No term of x^2 = $0x^2$

Cofficient of x is zero = No term of x = $0x$

3 (i) Find the first 3 terms in the expansion of $(2-x)^6$ in ascending powers of x. [3]

(ii) Find the value of k for which there is no term in x^2 in the expansion of $(1+kx)(2-x)^6$. [2]

(i) $(2-x)^6 = (2)^6 + {}^6C_1(2)^5(-x)^1 + {}^6C_2(2)^4(-x)^2$
$= 64 - 192x + 240x^2$

(ii) $(1+kx)(2-x)^6 \longrightarrow 0x^2$

× M1 $\quad 1 \times 240x^2 = 240\,x^2$

$+kx \times -192x = -192k\,x^2$

$\underline{0\,x^2}$

$240 - 192k = 0$

$240 = 192k$

$k = \frac{240}{192}$

$\boxed{k = 1.25}$

× A1

12 **(i)** Find the first three terms, in descending powers of x, in the expansion of $\left(x - \frac{2}{x}\right)^6$. [3]

(ii) Find the coefficient of x^4 in the expansion of $(1 + x^2)\left(x - \frac{2}{x}\right)^6$. [2]

(i) $\left(x - \frac{2}{x}\right)^6 = (x)^6 + {}^6C_1 (x)^5\left(\frac{-2}{x}\right)^1 + {}^6C_2 (x)^4\left(-\frac{2}{x}\right)^2$

$= x^6 + (6)(\cancel{x^5}^{\,4})\left(\frac{-2}{\cancel{x}}\right) + (15)(\cancel{x^4}^{\,2})\left(\frac{4}{\cancel{x^2}}\right)$

$= x^6 - 12x^4 + 60x^2$

(ii) $(1 + x^2)\left(x - \frac{2}{x}\right)^6 \longrightarrow x^4$

$1 \times -12x^4 = -12\,x^4$

$+x^2 \times 60x^2 = 60\,x^4$

$\underline{48\,x^4}$

10 **(i)** Find the first 3 terms in the expansion of $\left(2x - \frac{3}{x}\right)^5$ in ~~descending~~ powers of x. [3]

2 terms 6 terms

(ii) Hence find the coefficient of x in the expansion of $\left(1 + \frac{2}{x^2}\right)\left(2x - \frac{3}{x}\right)^5$. [2]

(i) $(2x)^5 + {}^5C_1 (2x)^4\left(-\frac{3}{x}\right)^1 + {}^5C_2 (2x)^3\left(\frac{-3}{x}\right)^2$

$(5)(16x^4)\left(\frac{-3}{x}\right) \qquad (10)(8x^3)\left(\frac{9}{x^2}\right)$

$\boxed{32x^5 - 240x^3 + 720x}$

(ii) $1 \times 720x = 720\,x$

$+\frac{2}{x^2} \times -240x^3 = -480\,x$

$\underline{240\,x}$

14 **(i)** Find the first 3 terms in the expansion, in ~~ascending powers~~ of x, of $(1 - 2x^2)^8$. [2]

(ii) Find the coefficient of x^4 in the expansion of $(2 - x^2)(1 - 2x^2)^8$. [2]

(i) $(1-2x^2)^8 = (1)^8 + {}^8C_1 (1)^7(-2x^2)^1 + {}^8C_2 (1)^6(-2x^2)^2$

$= 1 - 16x^2 + 112x^4$

(ii) $(2 - x^2)(1 - 2x^2)^8 \longrightarrow x^4$

$2 \times 112x^4 = 224x^4$

$-x^2 \times -16x^2 = 16x^4$

$240x^4$

11 (i) Find the first 3 terms in the expansion of $(1 + ax)^5$ in ascending powers of x. [2]

(ii) Given that there is no term in x in the expansion of $(1 - 2x)(1 + ax)^5$, find the value of the constant a. [2]

$0x$

(iii) For this value of a, find the coefficient of x^2 in the expansion of $(1 - 2x)(1 + ax)^5$. [3]

(i) $(1)^5 + {}^5C_1 (1)^4 (ax)^1 + {}^5C_2 (1)^3 (ax)^2$

$\boxed{1 + 5ax + 10a^2x^2}$

$1 + 5\left(\frac{2}{5}\right)x + 10\left(\frac{2}{5}\right)^2 x^2$

$1 + 2x + \frac{8}{5}x^2$

(ii) $(1-2x)(1+ax)^5 \longrightarrow 0x$

$1 \times 5ax = 5a\,x$

$-2x \times 1 = -2\,x$

$0\,x$

$5a - 2 = 0$

$\boxed{a = \frac{2}{5}}$

(iii) $(1-2x)(1+ax)^5 \longrightarrow x^2$

$1 \times 10a^2x^2 = 10a^2\,x^2$

$-2x \times 5ax = -10a\,x^2$

$(10a^2 - 10a)x^2$

$10a^2 - 10a$

$10\left(\frac{2}{5}\right)^2 - 10\left(\frac{2}{5}\right)$

$= \boxed{-2.4x^2}$

HOW TO EXTRACT ONE PARTICULAR TERM.

$\boxed{\text{Term NO}}$ $\boxed{T_{r+1} = {}^nC_r\, a^{n-r}\, b^r}$ $\boxed{\text{POWER OF } x}$

Q find 5th term in expansion of $(2-x)^6$

$T_{r+1} = {}^6C_r (2)^{6-r} (-x)^r$

$\boxed{r=4}$

$T_5 = {}^6C_4 (2)^{6-4} (-x)^4$

$= (15)(4)(x^4)$

$\boxed{T_5 = 60x^4}$

Q Find the term of x^2 in expansion of $(3x-2)^6$

$T_{r+1} = {}^6C_r (3x)^{6-r} (-2)^r$

$= {}^6C_r \cdot 3^{6-r} \cdot x^{6-r} \cdot (-2)^r$

$x^{6-r} = x^2$

$6 - r = 2$

$6 = r + 2$

$\boxed{r = 4}$

$T_{4+1} = {}^6C_4 (3x)^{6-4} (-2)^4$

$= (15)(3x)^2(16)$

$= (15)(9x^2)(16)$

$= \boxed{2160x^2}$

STEPS

1) Expand all powers and isolate x terms

2) Equate isolated x-terms to your required term. Find $\boxed{r}$

3) Put value of r in first step.

$\boxed{r}$ cannot be decimal or fraction or negative.

Q Find the term of x^2 in expansion of $\left(3x + \frac{2}{x}\right)^8$

$T_{r+1} = {}^8C_r (3x)^{8-r} \left(\frac{2}{x}\right)^r$

${}^8C_r \cdot 3^{8-r} \cdot x^{8-r} \cdot \frac{2^r}{x^r}$

$\frac{x^{8-r}}{x^r} = x^2$

$x^{8-2r} = x^2$

$8 - 2r = 2$

$6 = 2r$

$\boxed{r = 3}$

$T_{3+1} = {}^8C_3 (3x)^{8-3} \left(\frac{2}{x}\right)^3$

$T_4 = (56)(243\, \cancel{x^5}^{\,2}) \left(\frac{8}{\cancel{x^3}}\right)$

$T_4 = 108864\, x^2$

5 Find the coefficient of x^2 in the expansion of $\left(x + \frac{2}{x}\right)^6$. [3]

$T_{r+1} = {}^6C_r (x)^{6-r} \left(\frac{2}{x}\right)^r$

$= {}^6C_r \cdot x^{6-r} \cdot \frac{2^r}{x^r}$

$\frac{x^{6-r}}{x^r} = x^2$

$x^{6-r-r} = x^2$

$6 - 2r = 2$

$4 = 2r$

$\boxed{r = 2}$

$T_{2+1} = {}^6C_2 (x)^{6-2} \left(\frac{2}{x}\right)^2$

$= 15 \cdot \cancel{x^4}^{\,2} \cdot \frac{4}{\cancel{x^2}}$

$= 60x^2$

Cofficient $= 60$.

7 Find the value of the coefficient of x^2 in the expansion of $\left(\frac{x}{2} + \frac{2}{x}\right)^6$. [3]

$T_{r+1} = {}^6C_r \left(\frac{x}{2}\right)^{6-r} \left(\frac{2}{x}\right)^r$

${}^6C_r \cdot \frac{x^{6-r}}{2^{6-r}} \cdot \frac{2^r}{x^r}$

$\frac{x^{6-r}}{x^r} = x^2$ required power.

$x^{6-2r} = x^2$

$6 - 2r = 2$

$T_{2+1} = {}^6C_2 \left(\frac{x}{2}\right)^{6-2} \left(\frac{2}{x}\right)^2$

$= (15) \left(\frac{\cancel{x^4}^{\,2}}{\cancel{16}_{\,4}}\right) \left(\frac{\cancel{4}}{\cancel{x^2}}\right)$

$= \frac{15}{4} x^2$

Cofficient $= \frac{15}{4} = 3.75$

$$x^{6-2r} = x^2$$
$$6 - 2r = 2$$
$$4 = 2r$$
$$\boxed{r = 2}$$

Cofficient $= \frac{15}{4} = 3.75$

$\sqrt{\frac{2}{3}} = 0.66661 \ldots$

16 Find the coefficient of x in the expansion of $\left(x + \frac{2}{x^2}\right)^7$. [3]

$$T_{r+1} = {}^7C_r (x)^{7-r} \left(\frac{2}{x^2}\right)^r$$
$$= {}^7C_r \cdot x^{7-r} \cdot \frac{2^r}{x^{2r}}$$
$$\frac{x^{7-r}}{x^{2r}} = x$$
$$x^{7-r-2r} = x^1$$
$$7 - 3r = 1$$
$$6 = 3r$$
$$\boxed{r = 2}$$

$$T_{2+1} = {}^7C_2 (x)^{7-2} \left(\frac{2}{x^2}\right)^2$$
$$= (21)\, x^{\cancel{5}\,1} \cdot \frac{4}{\cancel{x^4}}$$
$$= 84x$$

Cofficient $= 84$

22 Find the coefficient of x^6 in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^7$. [4]

$$T_{r+1} = {}^7C_r (2x^3)^{7-r} \left(-\frac{1}{x^2}\right)^r$$
$$= {}^7C_r \cdot 2^{7-r}\, x^{3(7-r)} \cdot \frac{(-1)^r}{(x^2)^r}$$
$$\frac{x^{21-3r}}{x^{2r}} = x^6$$
$$x^{21-5r} = x^6$$
$$21 - 5r = 6$$
$$5r = 15$$
$$\boxed{r = 3}$$

$$T_{3+1} = {}^7C_3 (2x^3)^{7-3} \left(-\frac{1}{x^2}\right)^3 \rightarrow \frac{(-1)^3}{(x^2)^3}$$
$$(2)^4 (x^3)^4$$
$$= (35)(16x^{12}) \frac{-1}{x^6}$$
$$= -560x^6$$

27 Find the coefficient of x^3 in the expansion of $(2 - \frac{1}{2}x)^7$. [3]

$$T_{r+1} = {}^7C_r (2)^{7-r} \left(-\frac{1}{2}x\right)^r$$
$$= {}^7C_r \cdot 2^{7-r} \cdot \frac{(-1)^r}{2^r} \cdot x^r$$

r 3

$$T_{3+1} = {}^7C_3 (2)^4 \left(-\frac{1}{2}x\right)^3$$
$$= (35)(16) \left(\frac{-1x^3}{8}\right)$$
$$= -70x^3$$

2^r $= -70x^3$

$x^r = x^3$

$\boxed{r=3}$

NEW VOCABULARY:

Term Independent of $x \longrightarrow x^0$

15 Find the term independent of x in the expansion of $\left(x - \frac{1}{x^2}\right)^9$. [3]

Required $= x^0$

$T_{r+1} = {}^9C_r (x)^{9-r} \left(\frac{-1}{x^2}\right)^r$

$= {}^9C_r \cdot x^{9-r} \cdot \frac{(-1)^r}{x^{2r}}$

$\frac{x^{9-r}}{x^{2r}} = x^0 \longrightarrow$ Term independent of x. $\longrightarrow$

$x^{9-3r} = x^0$

$9 - 3r = 0$

$\boxed{r=3}$

$T_{3+1} = {}^9C_3 (x)^{9-3} \left(\frac{-1}{x^2}\right)^3$

$= 84 \cancel{x^6} \frac{(-1)}{\cancel{x^6}}$

$= \boxed{-84}$

19 Find the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^6$. [3]

x^0

$T_{r+1} = {}^6C_r (2x)^{6-r} \left(\frac{1}{x^2}\right)^r$

$= {}^6C_r \cdot 2^{6-r} \cdot x^{6-r} \cdot \frac{(1)^r}{(x^2)^r}$

$\frac{x^{6-r}}{x^{2r}} = x^0$

$x^{6-3r} = x^0$

$6 - 3r = 0$

$\boxed{r=2}$

$T_{2+1} = {}^6C_2 (2x)^4 \left(\frac{1}{x^2}\right)^2$

$(15)(16x^4)\left(\frac{1}{x^4}\right)$

$= \boxed{240}$

ZAINEMATICS

FOR THE LOVE OF MATHS

Sequences

P1

Compiled by Rafay Mushtaq

zainematics zainematics zainematics

SEQUENCES (AP/GP)

Tuesday, 4 August 2020 7:04 PM

Very Easy. (7 MARKS).

(Nothing Like O Levels)

ARITHEMATIC :- (AP)

$1, 4, 7, 10, -, -$ (+3, +3, +3) $n=4$ $\quad d=\frac{10-7}{7-4}=3$

$60, 55, 50 \ldots$ (−5, −5) $\quad d=\frac{50-55}{55-60}=-5$

$$\boxed{\text{Common difference} = d = \text{Next Term} - \text{Previous Term}}$$

FORMULAS.

1. nth term $= a + (n-1)d$

First term (a). Term No. (n). Common difference (d).

2. Sum to first n terms

$$S_n = \frac{n}{2}(a + l)$$

a = First term; l = Last term (if given).

3. If last term is not given, then formula for Sum is:

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

Q. $4, 7, 10, 13, \ldots, 61$ (+3, +3, +3), $n=20$ $\quad \boxed{AP} \quad a=4, \; d=13-10=+3$

(i) Find the 20th term in this sequence.

n^{th} term $= a+(n-1)d$.

20^{th} term $= 4+(20-1)(+3)$

20^{th} term $= 61$

(ii) Find Sum of first 20 terms.

$$S_{20} = \frac{n}{2}(a + l)$$ (l = last term)

$$= \frac{20}{2}(4+61)$$

$$S_{20} = 650$$

(iii) Find Sum of first 30 terms in sequence.

$$S_{30} = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{30}{2}\left[2(4)+(30-1)(3)\right]$$

$$= 15(8+87)$$

$$S_{30} = 1425.$$

$$S_n = \frac{n}{2}(a + l)$$

$$n^{th}\text{ term} = a+(n-1)d.$$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

GEOMETRIC PROGRESSION (GP).

$$S_\infty = \frac{a}{1-r} = \frac{2}{1-2} = \boxed{-2} \text{ ABSURD}$$ Diverging.

$2, 4, 8, 16, 32, 64 \ldots$ (×2, ×2, ×2, ×2) $\quad r = \frac{64}{32}, \frac{16}{8} = \boxed{2}$

$9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ (÷3, ÷3, ÷3, ÷3, ÷3) $\quad r = \frac{3}{9}, \frac{1}{3} = \boxed{\frac{1}{3}}$

1. nth term $= a \cdot r^{n-1}$

Dot stands for multiply

2. SUM OF FIRST n-TERMS.

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{a(1-r^n)}{1-r}.$$

You have to pick formula for which

$4, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ $\quad r = \frac{3}{9}, \frac{1}{3} = \boxed{\frac{1}{3}}$

$\div 3 \quad \div 3 \quad \div 3 \quad \div 3 \quad \div 3$

COMMON RATIO $= r = \dfrac{\text{Next Term}}{\text{Previous Term}}$

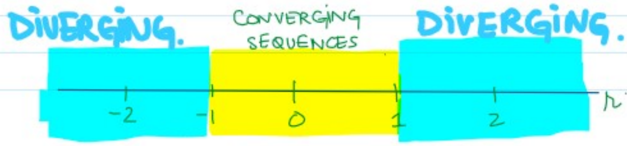
$r - 1 \qquad 1 - r$

You have to pick formula for which denominator is positive.

$r = 5$ (First) $\qquad r = \frac{1}{2}$ (second)

3 SUM TO INFINITY.

EXISTS ONLY FOR CONVERGING SEQUENCES.



IMP $\leftarrow$ $\boxed{-1 < r < 1}$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

If you apply this formula on a diverging sequence, It will given an answer but it will be absurd.

EXAM TRICK

	$a+(n-1)d$ AP	$a \cdot r^{n-1}$ GP
FIRST	a	a
SECOND	$a+d$	ar^1
THIRD	$a+2d$	ar^2
FOURTH	$a+3d$	ar^3
TENTH	$a+9d$	ar^9
100th Term	$a+99d$	ar^{99}
25th term	$a+24d$	ar^{24}

8th term of GP is 45

$ar^7 = 45$

15th term of AP is 70

$a + 14d = 70$

TYPE 1 = FORMULAS ONLY.

1 A geometric progression has first term 64 and sum to infinity 256. Find

(i) the common ratio, (r) [2]

(ii) the sum of the first ten terms. [2]

GP

$a = 64 \qquad S_\infty = 256$

(i) $S_\infty = \dfrac{a}{1-r}$

$256 = \dfrac{64}{1-r}$

(ii) $S_{10} = \dfrac{a(r^n - 1)}{r-1} = \dfrac{a(1-r^n)}{1-r}$

$\boxed{r = \frac{3}{4}}$ $\rightarrow$ +ve denominator.

$$256 = \frac{64}{1-r}$$
$$1-r = \frac{64}{256}$$
$$1-r = \frac{1}{4}$$
$$\boxed{r = \frac{3}{4}}$$

$\boxed{r=\frac{3}{4}}$ +ve denominator.

$$S_{10} = \frac{64(1-0.75^{10})}{1-0.75}$$
$$= 241.5837.$$

(b) A geometric progression has a common ratio of $-\frac{2}{3}$ and the sum of the first 3 terms is 35. Find

(i) the first term of the progression, [3]

(ii) the sum to infinity. [2]

$\boxed{GP}$

$r = -\frac{2}{3}$

$S_n = \frac{a(r^n-1)}{r-1}, \frac{a(1-r^n)}{1-r}$

$S_3 = 35$

$$S_3 = \frac{a(1-r^n)}{1-r}$$
$$35 = \frac{a\left(1-\left(\frac{-2}{3}\right)^3\right)}{1-\left(\frac{-2}{3}\right)}$$
$$\frac{175}{3} = a\left(\frac{35}{27}\right)$$
$$\boxed{a=45}$$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{45}{1-\left(-\frac{2}{3}\right)}$$
$$\boxed{S_\infty = 27}$$

14 The first term of a geometric progression is 12 and the second term is -6. Find

(i) the tenth term of the progression, [3]

(ii) the sum to infinity. [2]

$\boxed{GP}$ $a = 12$, Second $ar = -6$

$12r = -6$

$\boxed{r = -\frac{1}{2}}$

(i) Tenth term $= ar^9 = (12)\left(-\frac{1}{2}\right)^9 = 12\left(\frac{-1}{512}\right)$

$$= -\frac{3}{128}$$

(ii) $S_\infty = \frac{a}{1-r} = \frac{12}{1-\left(\frac{-1}{2}\right)} = \boxed{8}$

2 Find

(i) the sum of the first ten terms of the geometric progression 81, 54, 36, ..., [3]

32^{nd} term.

(ii) the sum of all the terms in the arithmetic progression 180, 175, 170, ..., 25. [3]

$\boxed{AP}$ 180, 175, 170, ..., 25

(ii) the sum of all the terms in the arithmetic progression 180, 175, 170, ..., 25. [3]

(i) GP

81, 54, 36 ....

$a = 81$, $r = \frac{54}{81} = \frac{2}{3}$

$S_{10} = \frac{a(1-r^n)}{1-r}$

$= \frac{81\left(1-\left(\frac{2}{3}\right)^{10}\right)}{1-\frac{2}{3}}$

$S_{10} = 238.786$.

AP 180, 175, 170 25

$a = 180$, $d = -5$

$S_n = \frac{n}{2}(a+l)$

$= \frac{32}{2}(180+25)$

$= 3280$

For n, apply nth term formula on last term.

nth term $= a+(n-1)d$

$25 = 180 + (n-1)(-5)$

$-155 = (-5)(n-1)$

$31 = n-1$

$\boxed{n = 32}$

6 **(a)** Find the sum of all the integers between 100 and 400 that are divisible by 7. [4]

(b) The first three terms in a geometric progression are 144, x and 64 respectively, where x is positive. Find

(i) the value of x,

(ii) the sum to infinity of the progression.

[5]

If this was AP, then we find Common difference twice and put it equal.

d = Next − prev.

~~100~~ ~~101~~ ~~400~~

105, 112, 119 399

$S_n = \frac{n}{2}(a+l)$

$= \frac{43}{2}(105+399)$

$= \boxed{10836}$

nth term $= a+(n-1)d$.

$399 = 105 + (n-1)(7)$

$294 = 7(n-1)$

$42 = n-1$

$n = 43$

(b) GP 144, x, 64

Common ratio stays same.

Find (r) twice and put it equal.

$\frac{x}{144} = \frac{64}{x}$

$x^2 = 64 \times 144$

$x = \sqrt{64 \times 144}$

$x = 96$

144, 96, 64

$r = \frac{96}{144}$

$S_\infty = \frac{a}{1-r} = \frac{144}{1-\frac{96}{144}}$

$= \boxed{432}$

7 The second term of a geometric progression is 3 and the sum to infinit is 12.

(i) Find the first term of the progression. [4]

An arithmetic progression has the same first and second terms as the geometric progression.

(ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

Note: First write in words, Then put all available values The equate.

(i) GP

second Term = 3

$ar = 3$

$r = \frac{3}{a}$

Sum to infinty = 12

$\frac{a}{1-r} = 12$

$a = 12\left(1-\frac{3}{a}\right)$

	The GP	An AP
First	= 6	First = a
Second	= 3	Second = $a+d$

$r = \frac{3}{a}$ $\quad a = 12\left(1 - \frac{3}{a}\right)$

$r = \frac{3}{6}$ $\quad a = 12 - \frac{36}{a}$

$\boxed{r = \frac{1}{2}}$

$a^2 = 12a - 36$

$a^2 - 12a + 36 = 0$

$a^2 - 6a - 6a + 36 = 0$

$a(a-6) - 6(a-6) = 0$

$(a-6)(a-6) = 0$

$\boxed{a = 6}$

Second = 3 $\quad$ Second = $a+d$

$a = 6,\quad a+d = 3$

$6 + d = 3$

$\boxed{d = -3}$

AP $\quad a = 6,\ d = -3$

$S_{20} = \frac{20}{2}\left[2(6) + (20-1)(-3)\right] = \boxed{-450}$

9 The first term of a geometric progression is 81 and the fourth term is 24. Find

(i) the common ratio of the progression, [2]

(ii) the sum to infinity of the progression. [2]

The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression.

(iii) Find the sum of the first ten terms of the arithmetic progression. [3]

GP $\quad$ First term = 81 $\quad$ Fourth term = 24

(i) $\boxed{a = 81}$

$ar^3 = 24$

$81 r^3 = 24$

$r^3 = \frac{8}{27}$

$\boxed{r = \frac{2}{3}}$

(ii) $S_\infty = \frac{a}{1-r} = \frac{81}{1-\frac{2}{3}} = \boxed{243}$

This Gp $\quad$ An AP

Second = $ar = (81)\left(\frac{2}{3}\right)$ $\quad$ First = a

Third = $ar^2 = (81)\left(\frac{2}{3}\right)^2$ $\quad$ Fourth = $a + 3d$

Do not put equal before putting all known values.

$81\left(\frac{2}{3}\right) = a$ $\quad 81\left(\frac{2}{3}\right)^2 = a + 3d$

$\boxed{a = 54}$ $\quad 81\left(\frac{4}{9}\right) = 54 + 3d$

$-18 = 3d$

$\boxed{d = -6}$

$S_{10} = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{10}{2}\left[2(54) + (10-1)(-6)\right]$

$= 270$

26 The first term of an arithmetic progression is 12 and the sum of the first 9 terms is 135.

(i) Find the common difference of the progression. [2]

The first term, the ninth term and the nth term of this arithmetic progression are the first term, the second term and the third term respectively of a geometric progression.

(ii) Find the common ratio of the geometric progression and the value of n. [5]

AP $\quad$ First = 12 $\quad S_9 = 135$

$a = 12$ $\quad \frac{9}{2}\left[2(12) + (9-1)d\right] = 135$

$24 + 8d = 30$

$8d = 6$

$\boxed{d = \frac{3}{4}}$

THIS AP $\quad$ ANOTHER GP

FIRST = $a = \boxed{12}$ $\quad$ FIRST = a

NINTH = $a + 8d = \boxed{12 + 8\left(\frac{3}{4}\right)}$ $\quad$ SECOND = ar

nth term = $a + (n-1)d$ $\quad$ THIRD = ar^2

$\boxed{12 + (n-1)\left(\frac{3}{4}\right)}$

$\boxed{a = 12}$ $\quad 12 + \cancel{8}^{2}\left(\frac{3}{\cancel{4}}\right) = ar$

$18 = 12r$

$12 + (n-1)\left(\frac{3}{4}\right) = ar^2$

$12 + (n-1)\left(\frac{3}{4}\right) = 12\left(\frac{3}{2}\right)^2$

$\boxed{a = \frac{\ }{4}}$ $\quad$ $\boxed{a = 12}$ $\quad$ $12 + 8\left(\frac{3}{4}\right) = ar$ $\quad$ $12 + (n-1)\left(\frac{3}{4}\right) = 12\left(\frac{3}{2}\right)^2$

$18 = 12r$

$\boxed{r = \frac{3}{2}}$ $\quad$ $\boxed{n = 21}$

TYPE 2 :- Sum.

25 (a) In an arithmetic progression, the sum of the first n terms, denoted by S_n, is given by

$$S_n = n^2 + 8n.$$

Find the first term and the common difference. [3]

$S_n = n^2 + 8n$ $\qquad$ Method: First put $n = 1$

$n = 1, \quad S_1 = (1)^2 + 8(1) = 9 \quad \boxed{a = 9}$

S_1 means sum of one term
That means it is first term.

$n = 2, \quad S_2 = (2)^2 + 8(2) = 20$

S_2 means sum of first two terms.

S_2 = First Term + Second term.
$20 = 9$ + Second term.
Second term $= 11$.

$\boxed{AP}$ $\quad 9, 11 \ldots\ldots \quad d = \text{Next} - \text{prev} = 11 - 9 = \boxed{2}$

First term $= 9$
Common Difference $= d = 2$

32 (a) In an arithmetic progression, the sum, S_n, of the first n terms is given by $S_n = 2n^2 + 8n$. Find the first term and the common difference of the progression. [3]

$S_n = 2n^2 + 8n$

$n = 1, \quad S_1 = 2(1)^2 + 8(1) = 10$ $\qquad$ $S_1 = \boxed{\text{first term} = 10}$

$n = 2, \quad S_2 = 2(2)^2 + 8(2) = 24$ $\qquad$ S_2 = First term + Second Term.

$24 = 10$ + Second Term.

$\boxed{\text{Second Term} = 14}$

$AP = 10, 14 \ldots\ldots$

Common difference $= 14 - 10$
$= 4$

TYPE 3: SCENARIO BASED :-

LAYMAN APPROACH

Read as if you have no idea of AP/GP chapter.
Predict first three Terms by common Sense.

Day/year	n	Terms.	Check AP $d = \text{Next} - \text{Prev}$	Check GP $r = \frac{\text{Next}}{\text{Prev}}$ ..
2012	1			
2013	2			
2014	3			

2013	2	
2014	3	
2020	$\boxed{9}$	

(b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be \$4000 in 2012 and will increase by 5% each year. Calculate

(i) the value of the grant in 2022, [2]

(ii) the total amount the college will receive in the years 2012 to 2022 inclusive. [2]

Year	n	Terms	Check AP $d = \text{Next} - \text{Prev.}$	Check GP $r = \frac{\text{Next}}{\text{prev.}}$
2012	1	4000	200	$\frac{4200}{4000} = 1.05$
2013	2	4200	210	$\frac{4410}{4200} = 1.05$
2014	3	4410		
2022	11			

$4000 \times 5\% = 200$

(i) Grant in 2022 = 11th term of GP = $a \cdot r^{10}$ ($a \cdot r^{n-1}$)

$(4000)(1.05)^{10}$

$= \boxed{6515.5785.}$

(ii) $S_{11} = \dfrac{a(r^n - 1)}{r - 1}$

$= \dfrac{4000(1.05^{11} - 1)}{1.05 - 1}$

$= 56\,827.14865$

5 Each year a company gives a grant to a charity. The amount given each year increases by 5% of its value in the preceding year. The grant in 2001 was \$5000. Find

(i) the grant given in 2011, [3]

(ii) the total amount of money given to the charity during the years 2001 to 2011 inclusive. [2]

Years	n	Terms	Check AP $d = N - P$	Check GP $r = \frac{N}{P}$
2001	1	5000	250	$r = \frac{5250}{5000} = 1.05$
2002	2	5250	262.5	$= \frac{5512.5}{5250} = 1.05$
2003	3	5512.5		
2011	$\boxed{11}$			

(i) Grant in 2011 = 11th term = $a \cdot r^{10}$

$= (5000)(1.05)^{10}$

$= 8144.473$

18 A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.

Model 1: Increase the prize money by \$1000 each day.

Model 2: Increase the prize money by 10% each day.

On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity

(i) if Model 1 is used, [4]

(ii) if Model 2 is used. [3]

ZAINEMATICS

FOR THE LOVE OF MATHS

Quadratics

P1

Compiled by Rafay Mushtaq

zainematics zainematics zainematics

QUADRATICS (P1)

Saturday, October 17, 2020 2:12 AM

Links with: 1) COORDINATE (9+)
2) FUNCTIONS (11+)

QUADRATIC EQUATION.

Max power of $x = 2$

$y = 9x^2 + 3x + 5$
$y = 6x - 2x^2$
$y = 8x^2$

[1] STANDARD FORM $y = ax^2 + bx + c$

SHAPE (a) — y-intercept (+c)

[2] COMPLETED SQUARE / VERTEX FORM / CONICAL FORM $y = a(x-b)^2 + c$

SHAPE (a)

Min Value if a +ve
Max value if a -ve.

TURNING POINT = $x - b = 0$, $\boxed{x = b}$ $\boxed{y = c}$

[3] ROOT FORM $y = a(x-b)(x-c)$

SHAPE (a)

x-intercepts. (ROOTS) $x - b = 0$, $\boxed{x = b}$ $x - c = 0$, $\boxed{x = c}$

STANDARD TO COMPLETED SQUARE FORM.

Q Express $y = 4x^2 - 24x + 40$ in form $a(x-b)^2 + c$

Half

$4\left[x^2 - 6x + (3)^2 - (3)^2 + 10\right]$

$4\left[(x - 3)^2 - 9 + 10\right]$

$4\left[(x-3)^2 + 1\right] = 4(x-3)^2 + 4$
$a(x-b)^2 + c$

$\boxed{a = 4}$, $-3 = -b$, $\boxed{b = 3}$ $\boxed{c = 4}$

Q Express $2x^2 + 7x - 12$ in form $a(x-b)^2 + c$.

Q Express $2x^2+7x-12$ in form $a(x-b)^2+c$.

$$2\left[x^2+\frac{7}{2}x+\left(\frac{7}{4}\right)^2-\left(\frac{7}{4}\right)^2-6\right]$$

Half.

$$2\left[\left(x+\frac{7}{4}\right)^2-\frac{49}{16}-6\right]$$

$$2\left[\left(x+\frac{7}{4}\right)^2-\frac{145}{16}\right]$$

$$2\left(x+\frac{7}{4}\right)^2-\frac{145}{8}$$

$$a(x-b)^2+c$$

$\boxed{a=2}$ $\quad -b=\frac{7}{4}$ $\quad \boxed{b=-\frac{7}{4}}$ $\quad \boxed{c=\frac{-145}{8}}$

Q Express $y=3x^2-24x+5$ in form $a(x+b)^2-c$

$$3\left[x^2-8x+(4)^2-(4)^2+\frac{5}{3}\right]$$

Half

$$3\left[(x-4)^2-16+\frac{5}{3}\right]$$

$$3\left[(x-4)^2-\frac{43}{3}\right]$$

$$3(x-4)^2-43$$

$$a(x+b)^2-c$$

$a=3,\ \boxed{+b=-4}\quad -c=-43\quad \boxed{c=43}$

Q Express $4x^2-24x+44$ in form $(2x-a)^2+b$.

STEP1: Completed square form.

$$4\left[x^2-6x+(3)^2-(3)^2+11\right]$$

Half

$$4\left[(x-3)^2-9+11\right]$$

$$4\left[(x-3)^2+2\right]$$

$$4(x-3)^2+8$$

$$2^2(x-3)^2+8$$

$$\left[2(x-3)\right]^2+8$$

$$(2x-6)^2+8$$

$$(2x-a)^2+b$$

$-6=-a\quad +b=8$

$\boxed{a=6}\quad \boxed{b=8}$

Q Express $9x^2-36x+20$ in form $(3x+a)^2+b$

Q Express $9x^2 - 36x + 20$ in form $(3x+a)^2 + b$

$$9\left[x^2 - 4x + (2)^2 - (2)^2 + \frac{20}{9}\right]$$
$$9\left[(x-2)^2 - 4 + \frac{20}{9}\right]$$
$$9\left[(x-2)^2 - \frac{16}{9}\right]$$

$$9(x-2)^2 - 16$$
$$3^2(x-2)^2 - 16$$
$$[3(x-2)]^2 - 16$$
$$(3x-6)^2 - 16$$
$$(3x+a)^2 + b$$
$$a = -6, \quad b = -16.$$

STANDARD $\longrightarrow$ ROOT FORM.

$y = ax^2 + bx + c$ $\quad$ $y = a(x-b)(x-c)$

FACTORIZATION.

$$y = x^2 - 7x + 12$$
$$= x^2 - 3x - 4x + 12$$
$$x(x-3) - 4(x-3)$$
$$y = 1(x-3)(x-4)$$
$$y = a(x-b)(x-c)$$

$a = 1$ $\quad$ $-3 = -b$, $\boxed{b=3}$ $\quad$ $-4 = -c$, $\boxed{c=4}$

SKETCH

1. SHAPE (ALL FORMS)

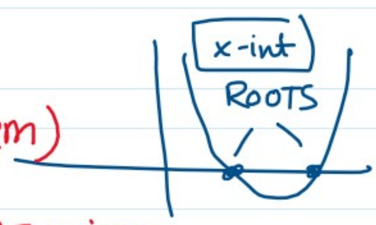
2. TURNING POINT (VERTEX) (COMPLETED SQUARE FORM).

$$y = 2(x-3)^2 - 4$$
$$x - 3 = 0$$

TURNING POINT $= \boxed{x=3}\ \boxed{y=-4} = (3, -4)$

3. y-intercept $(x=0)$ (STANDARD FORM)

$$y = 2x^2 + 7x + 1 \rightarrow \text{y intercept.}$$



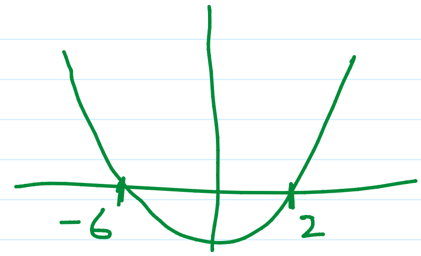
4. x-intercepts $(y=0)$ (ROOT FORM)

ONLY USE X-INTERCEPTS IF QUESTION REQUIRES.

$$y = 3(x-2)(x+6)$$

$$y = 3\underbrace{(x-2)}_{x-2=0}\underbrace{(x+6)}_{x+6=0}$$

X- INTERCEPTS $\boxed{x=2}$ $\boxed{x=-6}$



Q SKETCH: $y = 4x^2 - 24x + 40 \longrightarrow y = 4(x-3)^2 + 4$

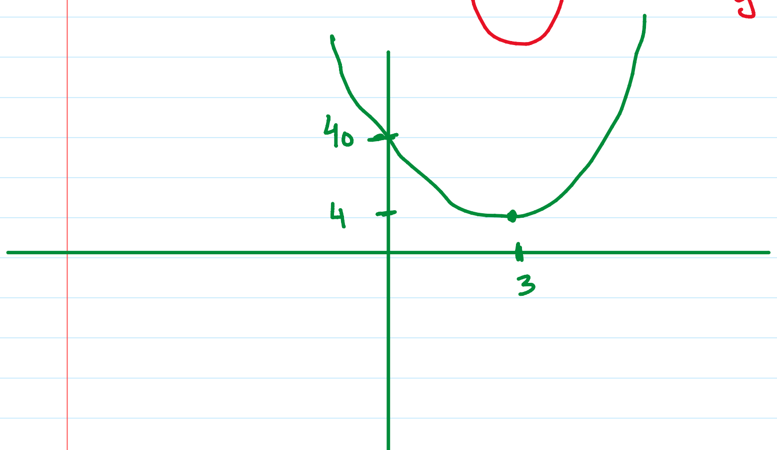
Shape

y-intercept

Shape

$x - 3 = 0$

Turning point. $\boxed{x=3}$ $\boxed{y=4}$



vertex form

$a(x-b)^2 + c$

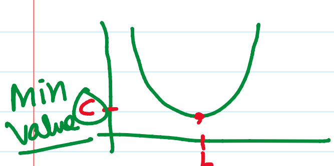
→ Min Value if a +ve

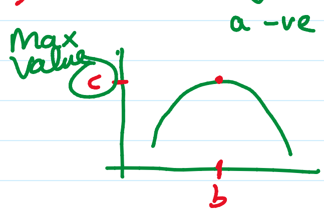
→ Max Value if a -ve.

TP = (b, c)

a +ve

a -ve





ROOTS (x-intercepts)

$y = 0$

$$y = 2x^2 - 3x + 8$$

$2x^2 - 3x + 8$ ... (Two Ans)

$y = 2x^2 - 3x + 8$

$0 = 2x^2 - 3x + 8$

$x = \dfrac{-b \pm \sqrt{\boxed{b^2-4ac}}}{2a}$

- +ve (Two Ans)
- Zero (1 Ans)
- −ve (No Ans)

$\boxed{\text{DISCRIMINANT}}$

$\boxed{b^2-4ac}$

positive (2 Ans)	Zero (1 Ans)	Negative (No Ans)

$b^2-4ac>0$	$b^2-4ac=0$	$b^2-4ac<0$
TWO DISTINCT REAL ROOTS	TWO EQUAL/REPEAT REAL ROOTS	NO REAL ROOT (Two Not Real Roots)

ONLY $\boxed{\text{Real}}$
- $b^2-4ac>0$
- $b^2-4ac=0$

$b^2-4ac \geq 0$

INEQUALITY

Q Find set of values of K for which $y = 3x^2 - 2x - 2K$ has no real roots

$a=3, \ b=-2, \ c=-2k$

$b^2-4ac<0$

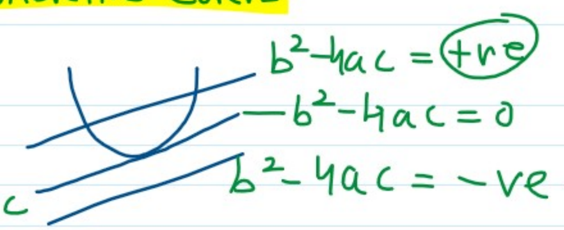
$$(-2)^2 - 4(3)(-2K) < 0$$
$$4 + 24K < 0$$
$$24K < -4$$
$$K < -\frac{4}{24}$$
$$\boxed{K < -\frac{1}{6}}$$

Q Find value of K for which $y = Kx^2 - 9x + 2$ has two real repeated roots.

$$b^2 - 4ac = 0$$
$$(-9)^2 - 4(K)(2) = 0$$
$$81 - 8K = 0$$
$$81 = 8K$$
$$\boxed{K = \frac{81}{8}}$$

INTERSECTION OF A LINE AND A QUADRATIC CURVE

$y = 2x^2 + 6x - 5$, $y = 2x + 1$



STEP 1: EQUATE BOTH

$b^2 - 4ac$

$$2x^2 + 6x - 5 = 2x + 1$$
$(4)^2 - 4(2)(-6)$

$+\ 64$ (Two Points of intersection).
$$2x^2 + 4x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

b^2-4ac:
- $+ve$ (Two Ans)
- $= 0$ (1 Ans)
- $-ve$ (No Ans)

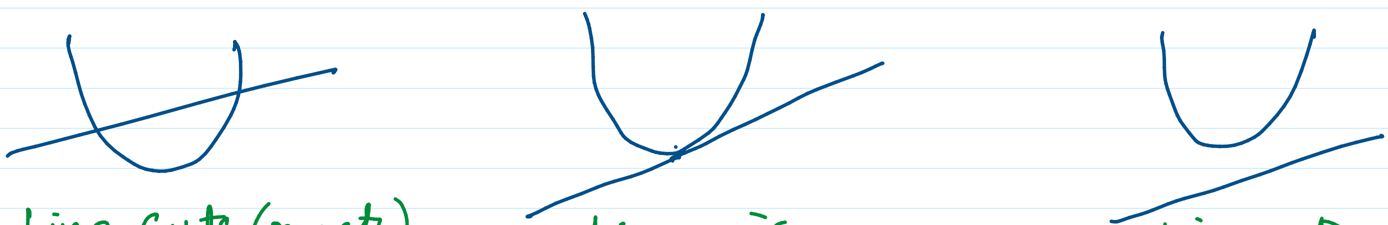
LINE & QUADRATIC CURVE

STEP1: EQUATE BOTH.

STEP2:

b^2-4ac

$b^2-4ac>0$	$b^2-4ac=0$	$b^2-4ac<0$



Line cuts (meets) curve at two points	LINE IS TANGENT TO CURVE	LINE DOES NOT INTERSECT THE CURVE.

INEQUALITIES

LINEAR

$\boxed{1}$ $2x-3>7$

$2x>10$

$x>5$

$\boxed{2}$ $-2x<6$

$x>\frac{6}{-2}$

$x>-3$

$\boxed{3}$ $\frac{x}{-3}>6$

$x<(6)(-3)$

$x<-18$.

QUADRATIC

$x^2>16$

YOU ARE NOT ALLOWED TO TAKE SQUARE ROOT ON AN INEQUALITY.

$(x-2)(x-5)>0$

$\cancel{x-2}>0$ $\quad$ $\cancel{x-5}>0$

YOU CANNOT DO EITHER/OR STEP ON AN INEQUALITY.

STEP1: FACTORIZE.

STEP2: SKETCH $\begin{cases}\text{SHAPE}\\ x\text{-intercepts}\end{cases}$.

STEP3: Colour the relevant part of curve.

STEP4: Write inequality for x-values of coloured parts of curve only.

$\underline{Q}$ $x^2-6x-16<0$

$x^2-8x+2x-16<0$

$x(x-8)+2(x-8)<0$

STEP1: FACTORIZE.

STEP2: SKETCH $\begin{cases}\text{SHAPE}\\ x\text{-intercepts}\end{cases}$

$x^2 - 8x + 2x - 16 < 0$

$x(x-8) + 2(x-8) < 0$

$\underbrace{(x-8)(x+2)}_{y<0 \text{ (below } x\text{-axis)}} < 0$

$\boxed{-2 < x < 8}$

STEP2 : SKETCH < SHAPE / x-intercepts

STEP3: Colour the relevant part of curve.

STEP4: Write inequality for x-values of coloured parts of curve only.

Q

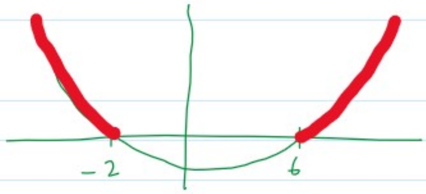
$x^2 - 4x - 12 > 0$

$x^2 - 6x + 2x - 12 > 0$

$x(x-6) + 2(x-6) > 0$

$(x-6)(x+2) > 0$

$y > 0$ (above x-axis)



$\boxed{x < -2}$ or $\boxed{x > 6}$

STEP1: FACTORIZE.

STEP2: SKETCH < SHAPE / x-intercepts

STEP3: Colour the relevant part of curve.

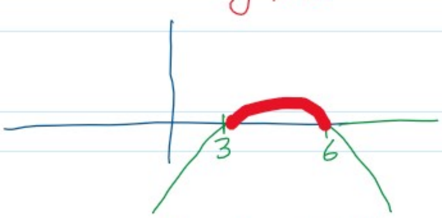
STEP4: Write inequality for x-values of coloured parts of curve only.

Q

$-x^2$

$(x-3)(6-x) > 0$

$y > 0$



$3 < x < 6$

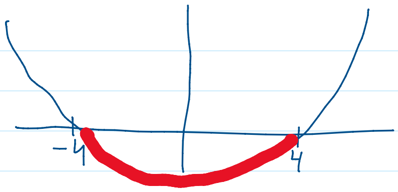
Q

$x^2 < 16$

$x^2 - 16 < 0$

$x^2 - 4^2 < 0$

$(x+4)(x-4) < 0$



$-4 < x < 4$

ZAINEMATICS

FOR THE LOVE OF MATHS

Functions

P1

Compiled by Rafay Mushtaq

zainematics zainematics zainematics

FUNCTIONS (P1)

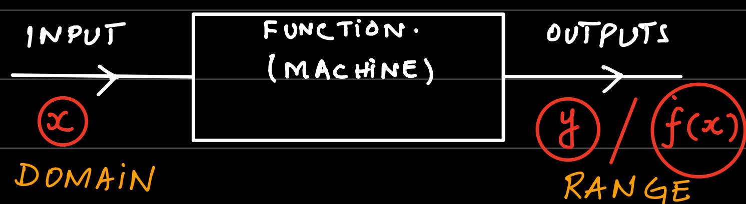
(10 MARKS)

FUNCTIONS ALONE	WITH QUADRATICS	WITH TRIG	WITH DIFFERENTIATION

VERY LONG BUT EASY CHAPTER.

MEMORIZE EVERY DETAIL!

FUNCTIONS ARE NUMBER MACHINES.



$f(x) = 2x + 3$ $\qquad$ $g(x) = x^2 - 5$

Name of function $\qquad$ value of input (x)

$f(1) = 2(1) + 3 = 5$

$f(4) = 2(4) + 3 = 11$

$g(3) = (3)^2 - 5 = 4$

$f(K) = 2K + 3$

$g(t-1) = (t-1)^2 - 5$

INVERSE OF A FUNCTION:

(2 Marks in O Levels) (3 MARKS IN A LEVELS)

THERE ARE ADDITIONAL STEPS. BE CAREFUL!

$$f(x) = 2x - 5$$

STEP 1: $f(x) = y$

$$y = 2x - 5$$

$$y + 5 = 2x$$

$f(x) = y$

STEP 2: $x = \dfrac{y+5}{2}$

$f^{-1}(y) = x$

STEP 3: $f^{-1}(y) = x$

STEP 4: $f^{-1}(y) = \dfrac{y+5}{2}$

$$f^{-1}(x) = \frac{x+5}{2}$$

Q: $f(x) = 2x + 8$

$$f(x) = y$$
$$y = 2x + 8$$
$$y - 8 = 2x$$
$$x = \frac{y-8}{2}$$

$$f^{-1}(y) = x$$

$$f^{-1}(y) = \frac{y-8}{2}$$

$$f^{-1}(x) = \frac{x-8}{2}$$

Q: $g(x) = x^3 - 8$

$$g(x) = y$$
$$y = x^3 - 8$$
$$y + 8 = x^3$$
$$x^3 = y + 8$$
$$x = \sqrt[3]{y+8}$$
$$g^{-1}(y) = x$$

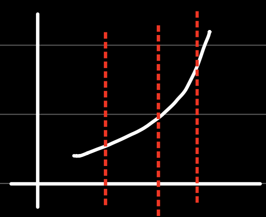
$$g^{-1}(y) = \sqrt[3]{y+8}$$

$$\boxed{g^{-1}(x) = \sqrt[3]{x+8}}$$

TEST

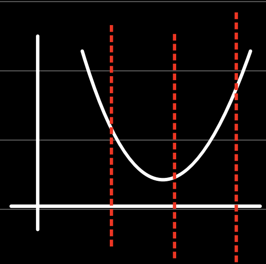
VERTICAL LINE TEST	HORIZONTAL LINE TEST
CHECKS WHETER A GRAPH IS A FUNCTION OR NOT?	CHECKS IF THE INVERSE OF A FUNCTION EXISTS OR NOT?

VERTICAL LINE TEST



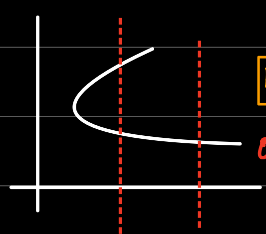
PASS IT IS FUNCTION

ONE - ONE FUNCTION



PASS IT IS FUNCTION

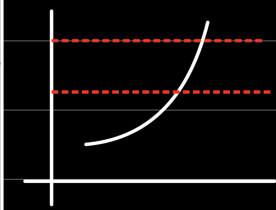
MANY - ONE FUNCTION



FAIL NOT A FUNCTION

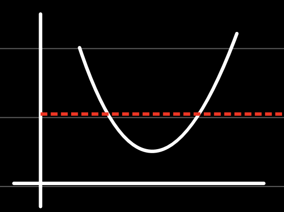
ONE - MANY.

HORIZONTAL LINE TEST



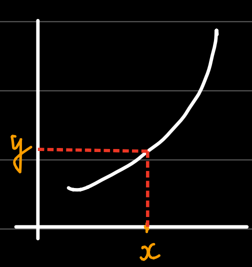
PASS INVERSE EXISTS

ONE - ONE FUNCTION



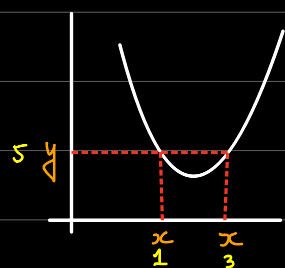
FAIL INVERSE DOES NOT EXIST.

MANY - ONE FUNCTION



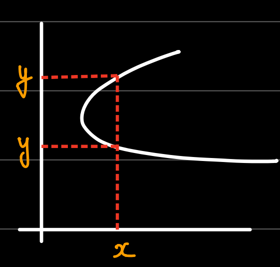
ONE - ONE FUNCTION

For one input you will get one output.



MANY TO ONE FUNCTION

For Many inputs you can get one output.

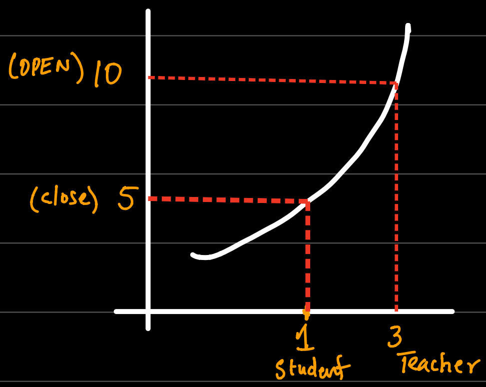


ONE TO MANY

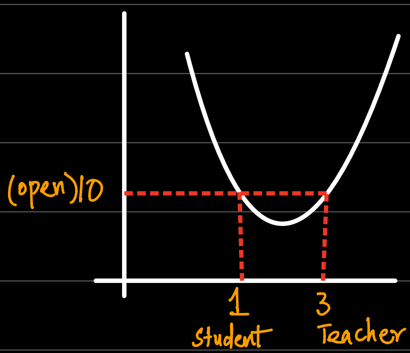
One input can get you many outputs.

ACCESS POINTS

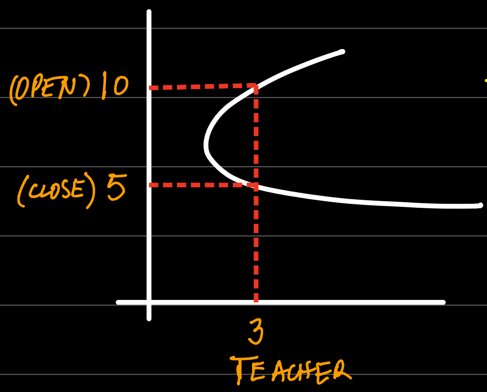
INPUT = 1 (STUDENT) OUTPUT = 5 (CLOSE)
= 3 (TEACHER) = 10 (OPEN)



→ This would be useful at STAFF ROOM DOOR.



→ THIS WOULD BE USEFUL AT SCHOOL MAIN GATE etc.



→ THIS is NOT A USEFUL MACHINE.

ONE-TO-MANY is NOT A FUNCTION.

COMPOSITE FUNCTIONS

(FUNCTION WITHIN A FUCTION)

$f(x) = 2x + 5$ $\qquad$ $g(x) = x^2 + 3$

$$fg(x) = f(g(x)) = 2\,g(x) + 5$$
$$= 2(x^2 + 3) + 5$$
$$= 2x^2 + 6 + 5$$
$$= 2x^2 + 11$$

ESSENTIAL WORKING.

$$gf(x) = (2x+5)^2 + 3$$

$$ff(x) = 2(2x+5) + 5$$

$$gg(x) = (x^2 + 3)^2 + 3$$

STEP 1: write the first function with ( ) instead of x.

STEP 2: Write value of second function inside the bracket.

Q: $\qquad f(x) = 2x + 3$

Given that $ff(t) = 21$, find t.

$$ff(x) = 2(2x+3) + 3 = 4x + 6 + 3 = 4x + 9$$
$$ff(x) = 4x + 9$$
$$ff(t) = 4t + 9$$
$$21 = 4t + 9$$

$4t = 12$

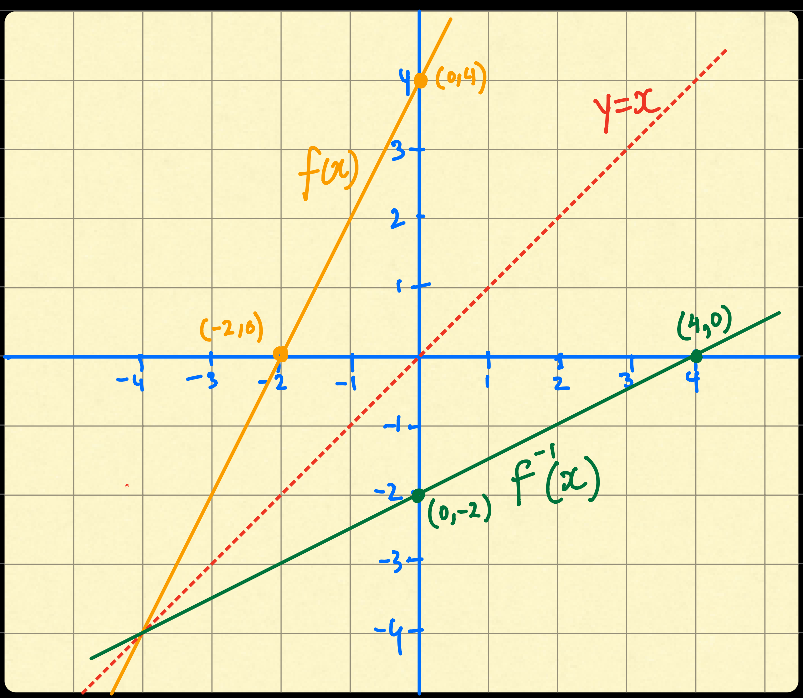
$t = 3$

SKETCH OF $f(x)$ and $f^{-1}(x)$ ON SAME DIAGRAM.

Q: $f(x) = 2x + 4$

Sketch $f(x)$ and $f^{-1}(x)$ on same diagram making clear the relationship between them. (3 marks).

IMP: THIS HAS TO BE DRAWN TO-SCALE.



$f(x) = 2x+4$

$y = 2x+4$

STRAIGHT LINE.

x-intercept $\quad y=0$

$0 = 2x+4$

$x = -2$

y-intercept $\quad x=0$

$y = 2(0)+4$

$y = 4$

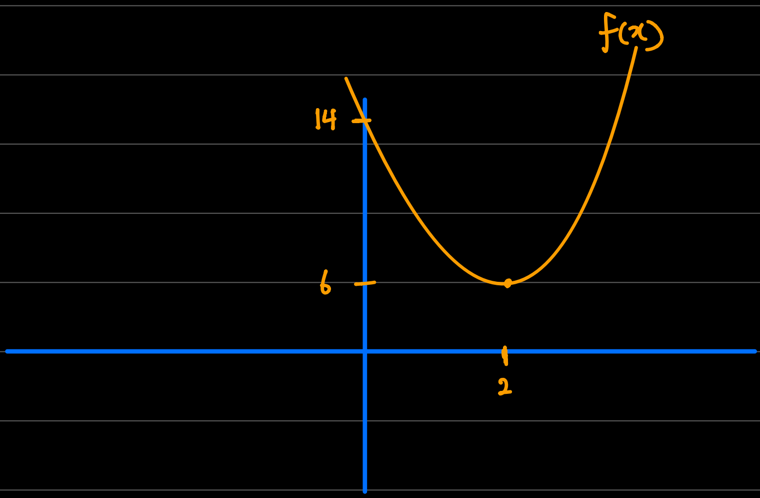
$f^{-1}(x)$ is reflection of $f(x)$ in line $y=x$.

Q: $f(x) = 2x^2 - 8x + 14$

(a) Express $f(x)$ in form $a(x-b)^2 + c$

$$2\left[x^2 - 4x + (2)^2 - (2)^2 + 7\right]$$
$$2\left[(x-2)^2 - 4 + 7\right]$$
$$2\left[(x-2)^2 + 3\right]$$
$$2(x-2)^2 + 6$$

(b) Sketch $f(x)$.



$f(x) = 2x^2 - 8x + 14$

Shape

y-intercept

$f(x) = 2(x-2)^2 + 6$

Shape

$x - 2 = 0$

Turning pt: $\boxed{x=2}$ $\boxed{y=6}$

(c) State with a reason whether $f^{-1}(x)$ exists or not?

No, inverse does not exist because it is not a one-one function.

$\boxed{\text{DOMAIN}}$ VALUES OF X FOR WHICH A FUNCTION DOES NOT GO CRAZY.

TYPE 1

$f(x) = \sqrt{x}$

Domain:
$x \geq 0$
$x \in R$

$f(x) = \sqrt{x-2}$

Domain:
$x \geq 2$
$x \in R$

GENERAL RULE

$f(x) = \sqrt{\square}$

$x \in R$

x — belongs to — Real Numbers — Any no.

Domain:
$\square \geq 0$ and solve.
$x \in R$

$f(x) = \sqrt{2x+3}$

Domain: $2x + 3 \geq 0$

$2x \geq -3$

$\boxed{x \geq -\frac{3}{2} \text{ and } x \in R}$

TYPE 2

$f(x) = \frac{3}{x}$

Domain: $x \neq 0$
$x \in R$

$f(x) = \frac{3}{x-2}$

Domain: $x \neq 2$
$x \in R$

GENERAL RULE.

$f(x) = \frac{\text{Anything}}{\square}$

Domain: $\square \neq 0$ solve
$x \in R$

$f(x) = \dfrac{2x+5}{\boxed{3x-1}}$

Domain:

$3x - 1 \neq 0$

$3x \neq 1$

$\boxed{x \neq \frac{1}{3} \quad \text{and} \quad x \in R}$

RANGE: VALUES OF Y THAT GRAPH OF $f(x)$ COVERS.

GOLDEN RULE: NEVER TELL RANGE OF A FUNCTION WITHOUT LOOKING AT ITS GRAPH

FIND RANGE OF FOLLOWING FUNCTIONS:

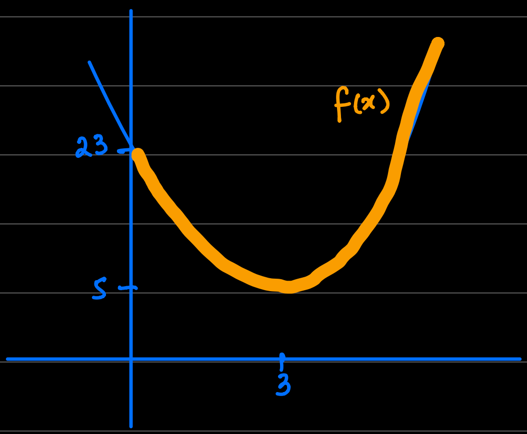
(a) $f(x) = 2(x-3)^2 + 5 \quad \underbrace{x \in R}$

you are allowed all values of x.

Range: $y \geq 5$

$f(x) \geq 5$

(b) $f(x) = 2(x-3)^2 + 5 \quad x > 0$



Range: $y \geq 5$

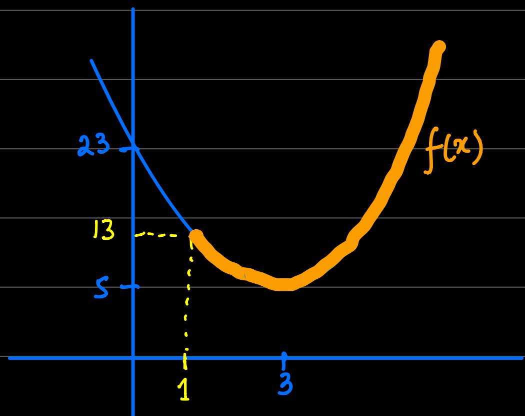
$f(x) \geq 5$

Range is usually written by name of function.

(c) $f(x) = 2(x-3)^2 + 5 \quad x \geqslant 1$

Put $x = 1$, $y = 2(1-3)^2 + 5$

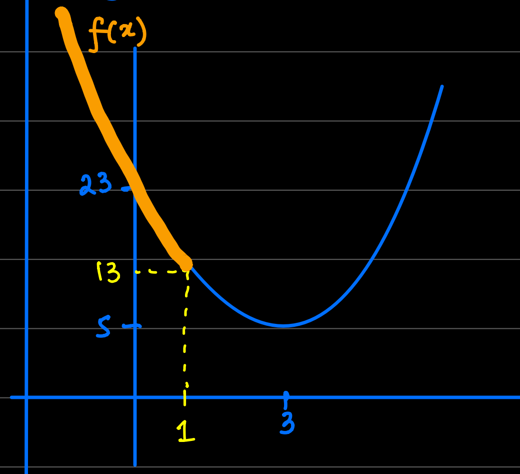
$y = 13$



Range: $y \geqslant 5$

$f(x) \geqslant 5$

(d) $f(x) = 2(x-3)^2 + 5 \quad x < 1$



Range: $y > 13$

$f(x) > 13$

(e) $f(x) = 2(x-3)^2 + 5 \quad x \geqslant 5$

Put $x = 5$, $y = 2(5-3)^2 + 5$

$y = 13$

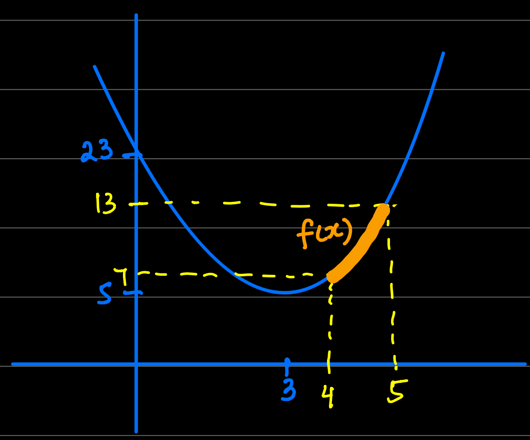
Range: $y \geqslant 13$

$f(x) \geqslant 13$

(f) $f(x) = 2(x-3)^2 + 5 \quad 4 < x \leqslant 5$

put $x = 4$, $y = 2(4-3)^2 + 5 =$

$y = 7$

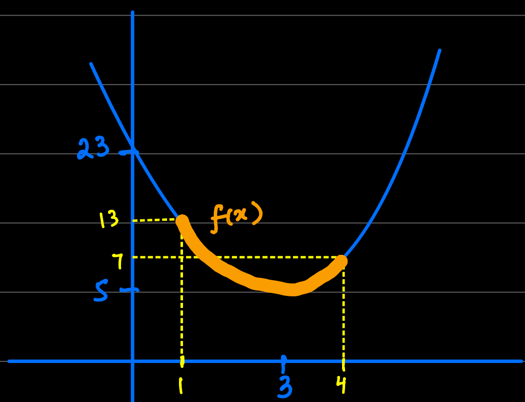


Range: $7 < y \leqslant 13$

$7 < f(x) \leqslant 13$

(g)

$f(x) = 2(x-3)^2 + 5 \quad 1 < x < 4$

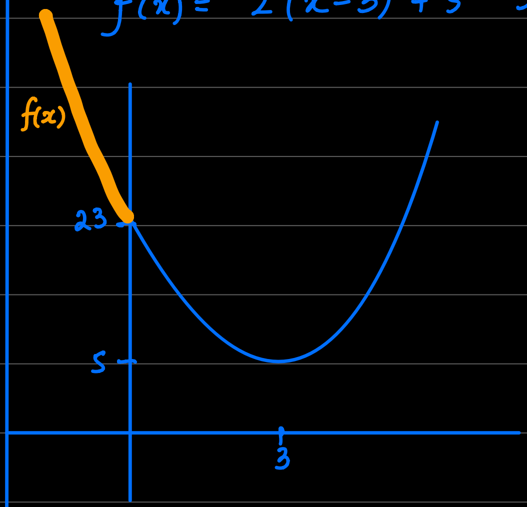


Range: $5 \leq y < 13$

$5 \leq f(x) < 13$

(h)

$f(x) = 2(x-3)^2 + 5 \quad x < 0$



Range: $y > 23$

$f(x) > 23$

ZAINEMATICS

FOR THE LOVE OF MATHS

Circular Measure

P1

Compiled by Rafay Mushtaq

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TRIGONOMETRY.

CIRCULAR MEASURE (8 MARKS)

PURE TRIGONOMETRY. (8 MARKS).

BASICS :-

UNITS

ANGLE.

DEGREES

RADIANS

$$180^\circ = \pi \text{ rad.}$$

Q Convert 110° to radians.

$180^\circ \longrightarrow \pi$ rad.

$110 \longrightarrow x$

$$180x = 110\pi$$

$$x = \frac{110\pi}{180}$$

$$x = 1.919 \text{ rad.}$$

Q Convert $\frac{3\pi}{2}$ rad to degrees

$180^\circ \longrightarrow \pi$ rad

$x \longrightarrow \frac{3\pi}{2}$

$$\pi x = 180 \times \frac{3\pi}{2}$$

$$\boxed{x = 270^\circ}$$

FAMOUS ANGLES

$180^\circ \longrightarrow \pi$ rad.

$90^\circ \longrightarrow \frac{\pi}{2}$

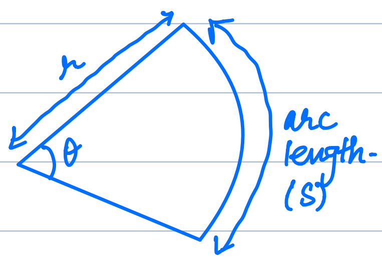
$45^\circ \longrightarrow \frac{\pi}{4}$

$30^\circ \longrightarrow \frac{\pi}{6}$

$60^\circ \longrightarrow \frac{\pi}{3}$

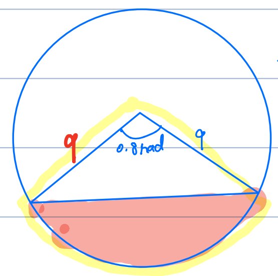
$360^\circ \longrightarrow 2\pi$

DEGREES	0	30	45	60	90
RADIANS -	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ infinite



DEGREE (θ)	RADIANS (θ)
Arclength $= \frac{\theta^\circ}{360} \times 2\pi r$	Arclength $S = r\theta$
Area of Sector $= \frac{\theta^\circ}{360} \times \pi r^2$	Area of Sector: $A = \frac{1}{2} r \underset{r\theta}{s}$

$A = \frac{1}{2} r^2 \theta$



Find Shaded area?

Shaded Area = Sector − Triangle

$\frac{1}{2}r^2\theta - \frac{1}{2}\underbrace{\square\,\square}_{\text{neighboring sides.}}\sin(\underset{\text{Angle}}{\bigcirc})$

CASIO fx 82MS
CASIO fx 350MS

$\frac{1}{2}(9)^2(0.8) - \frac{1}{2}\boxed{9}\,\boxed{9}\underbrace{\sin(0.8)}_{\text{calculator mode change.}}$

$= 32.4 - 29.05$

$= 3.35$

WHEN TO CHANGE CALCULATOR TO RADIAN MODE

ONLY WHEN

$\sin\square$	$\sin^{-1}\square$
$\cos\square$	$\cos^{-1}\square$
$\tan\square$	$\tan^{-1}\square$

AND ITS A RADIANS QUESTION.

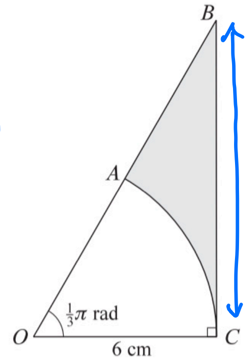
2

Shaded Area = Triangle − Sector

$\frac{1}{2}(OC)(BC) - \frac{1}{2}r^2\theta$

$\frac{1}{2}(6)(6\sqrt{3}) - \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$

$\boxed{18\sqrt{3} - 6\pi}$



$\tan\frac{\pi}{3} = \frac{BC}{6}$

$\sqrt{3} = \frac{BC}{6}$

$BC = 6\sqrt{3}$

6

$\sin\frac{\pi}{3} = \frac{OC}{12}$

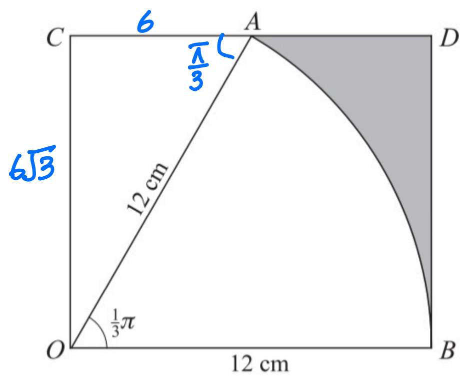
$\frac{\sqrt{3}}{2} = \frac{OC}{12}$

$\boxed{OC = 6\sqrt{3}}$

$\cos\frac{\pi}{3} = \frac{AC}{12}$

$\frac{1}{2} = \frac{AC}{12}$

$\boxed{AC = 6}$.



In the diagram, AOB is a sector of a circle with centre O and radius 12 cm. The point A lies on the side CD of the rectangle $OCDB$. Angle $AOB = \frac{1}{3}\pi$ radians. Express the area of the shaded region in the form $a(\sqrt{3}) - b\pi$, stating the values of the integers a and b. [6]

exact form.

SHADED AREA = RECTANGLE − TRIANGLE − SECTOR.

$(12)(6\sqrt{3}) - \frac{1}{2}(6)(6\sqrt{3}) - \frac{1}{2}(12)^2\left(\frac{\pi}{3}\right)$

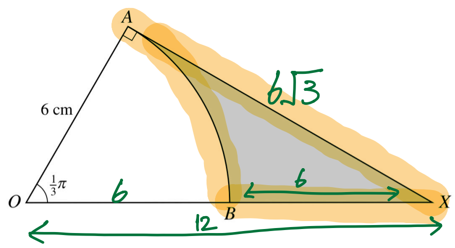
$= 72\sqrt{3} - 18\sqrt{3} - 24\pi$

$54\sqrt{3} - 24\pi$

$a\sqrt{3} - b\pi$

$\boxed{a=54}$, $-b=-24$

$\boxed{b=24}$



In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A, and OBX is a straight line.

(i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

Find, in terms of π and $\sqrt{3}$,

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [4]

$\tan\left(\frac{1}{3}\pi\right) = \frac{AX}{6}$

$\sqrt{3} = \frac{AX}{6}$

$\boxed{AX = 6\sqrt{3}}$

Area of shaded reigon = Triangle − sector

$\frac{1}{2}(OA)(AX) - \frac{1}{2}r^2\theta$

$\frac{1}{2}(6)(6\sqrt{3}) - \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$

$\boxed{18\sqrt{3} - 6\pi}$

Perimeter = AX + BX + Arc AB

$6\sqrt{3} + (6) + (2\pi)$

$= \boxed{6 + 6\sqrt{3} + 2\pi}$

$\boxed{BX}$

$OX^2 = OA^2 + AX^2$

$OX^2 = 6^2 + (6\sqrt{3})^2$

$= 36 + 108$

$\boxed{\text{Arc } AB}$

$S = r\theta$

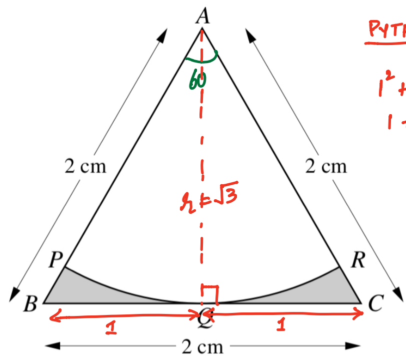
$= 6\left(\frac{\pi}{3}\right)$

$= 2\pi$

$Ox^2 = 144$

$OX = 12$

22



In the diagram, ABC is an equilateral triangle of side 2 cm. The mid-point of BC is Q. An arc of a circle with centre A touches BC at Q, and meets AB at P and AC at R. Find the total area of the shaded regions, giving your answer in terms of π and $\sqrt{3}$. [5]

Shaded Area = Triangle − Sector

$\frac{1}{\cancel{2}} \boxed{\cancel{2}} \boxed{2} \sin(60) - \frac{\theta}{360} \times \pi r^2$ $\theta = 60$, $r = \sqrt{3}$

$2\left(\frac{\sqrt{3}}{\cancel{2}}\right) - \frac{60}{360} \times \pi (\sqrt{3})^2$

$\sqrt{3} - \frac{1}{6}\pi(3)$

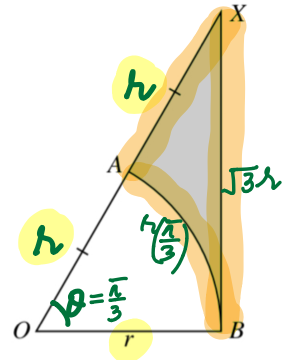
$= \boxed{\sqrt{3} - \frac{\pi}{2}}$

24

$\cos\theta = \dfrac{r}{2r}$

$\cos\theta = \dfrac{1}{2}$

$\boxed{\theta = \dfrac{\pi}{3}}$



PYTHAGORAS

$(2r)^2 = r^2 + BX^2$

$4r^2 = r^2 + BX^2$

$BX^2 = 3r^2$

$BX = \sqrt{3r^2} = \sqrt{3}\sqrt{r^2}$

$BX = \sqrt{3}r$

In the diagram, AB is an arc of a circle with centre O and radius r. The line XB is a tangent to the circle at B and A is the mid-point of OX.

Perimeter $= r + \sqrt{3}r + \dfrac{\pi r}{3}$

Arclength $AB = s = r\theta$
$= r\left(\dfrac{\pi}{3}\right)$

(i) Show that angle $AOB = \frac{1}{3}\pi$ radians. [2]

Express each of the following in terms of r, π and $\sqrt{3}$:

(ii) the perimeter of the shaded region, [3]

(iii) the area of the shaded region. = Triangle − Sector. [2]

$\frac{1}{2}(r)(r\sqrt{3}) - \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$

$$\boxed{\frac{r^2\sqrt{3}}{2} - \frac{r^2\pi}{6}}$$

ZAINEMATICS

FOR THE LOVE OF MATHS

Trignometry

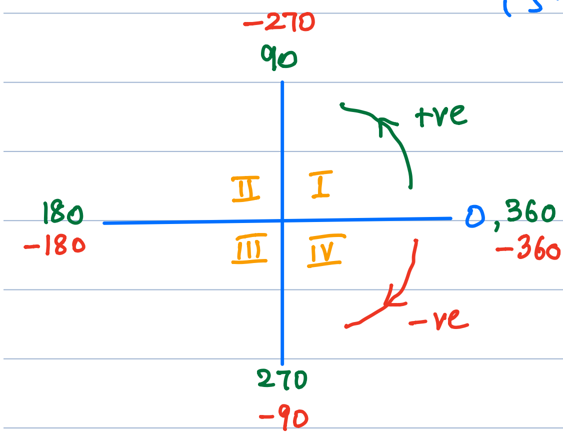
P1

Compiled by Rafay Mushtaq

zainematics zainematics zainematics

TRIGONOMETRY:

(5–8 Marks).



All Science Teachers Crazy

EVERY ANGLE HAS TWO MAIN VALUES.

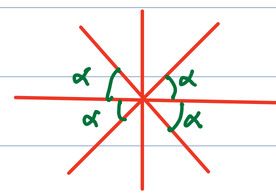
θ = ORIGINAL ANGLE

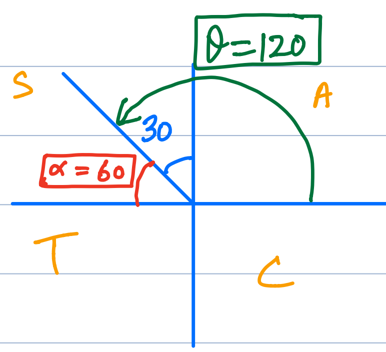
(STARTS FROM +ve x-axis)

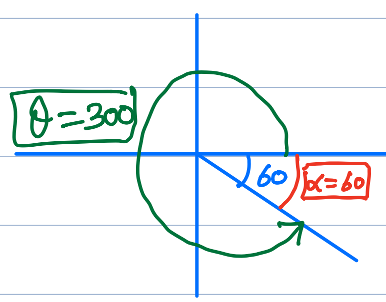
ACW (+ve) CW(-ve)

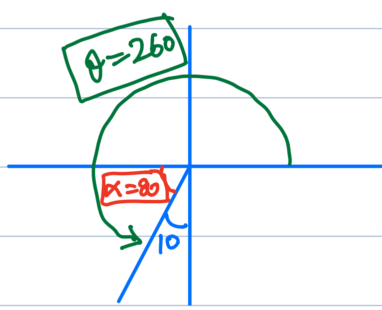
BASIC ANGLE: (α)

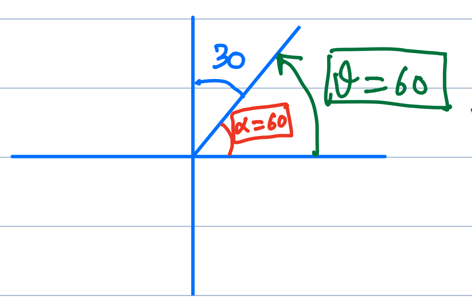
ACUTE ANGLE MADE WITH x-axis.











Sin/cos/tan.

[TRIG RATIO OF ANY ANGLE (θ)] = [TRIG RATIO OF ITS BASIC ANGLE (α) AFTER ADJUSTING QUADRANT +/- SIGN.]

Q. WITHOUT USING CALCULATOR EVALUATE.

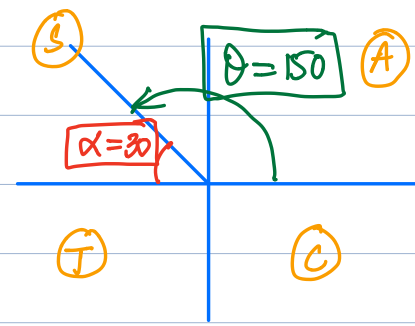
(i) cos 150

$\downarrow$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\downarrow$

$\boxed{\cos 150 = -\frac{\sqrt{3}}{2}}$



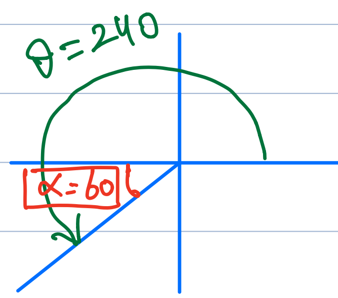
2 tan 240

$\downarrow$

$\tan 60 = \sqrt{3}$

$\downarrow$

$\tan 240 = +\sqrt{3}$



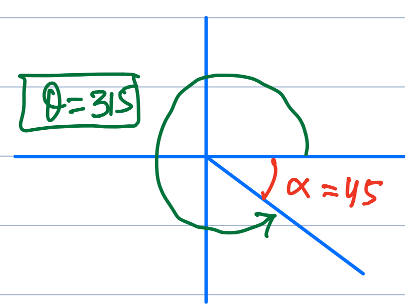
3 Sin 315

$\downarrow$

$\sin 45 = \frac{1}{\sqrt{2}}$

$\downarrow$

$\boxed{\sin 315 = -\frac{1}{\sqrt{2}}}$



EQUATION SOLVING.

① permission to take inverse

QUADRANT

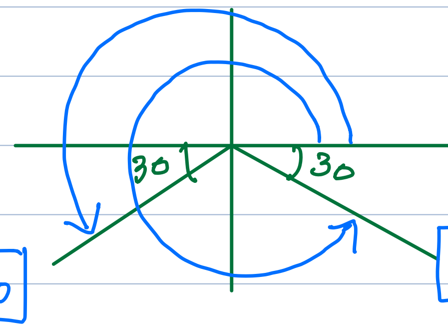
II. $\sin x = -\frac{1}{2}$ $\qquad 0 < x < 360$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

③ Basic Angle.

② ignore +/− sign while taking inverse

$\alpha = 30^\circ$



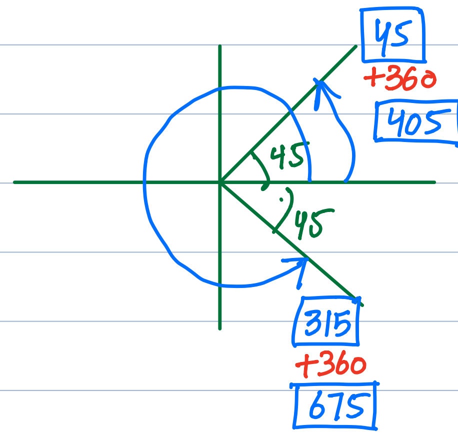
$x = 210, 330$

$\boxed{2}$ $\cos x = -\frac{1}{\sqrt{2}}$ $0 < x < 720$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\alpha = 45°$

$x = 45, 315, 405, 675$

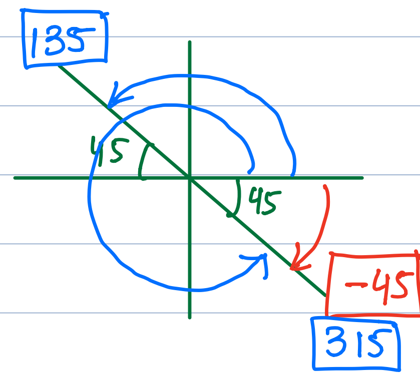


$\boxed{3}$ $\tan x = -1$ $-180 < x < 360$

$\alpha = \tan^{-1}(1)$

$\alpha = 45°$

$\boxed{x = -45, 135, 315}$

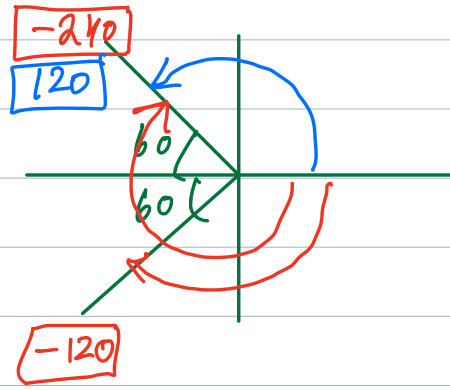


<u>NOTE:</u> IF YOUR RANGE IS IN NEGATIVE ALWAYS FIND NEGATIVE ANGLES FIRST.

$\boxed{4}$

$\cos x = -\frac{1}{2}$ $\qquad -360 < x < 180$

$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60$



$x = -240, -120, 120$

IF WE DO NOT HAVE PERMISSION TO TAKE INVERSE : RANGE CHANGE.

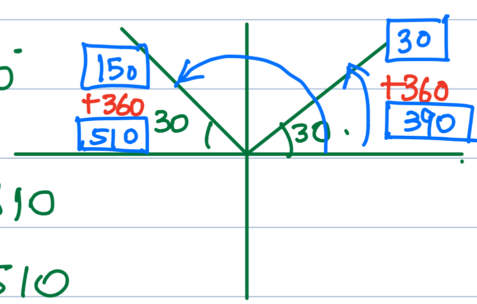
$\sin 2x = \frac{1}{2}$ $\qquad 0 < x < 360$

$\times 2$

$\boxed{2x = A}$ $\qquad 0 < 2x < 720$

$\sin A = \frac{1}{2}$ $\qquad 0 < A < 720$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$



$A = 30, 150, 390, 510$

$2x = 30, 150, 390, 510$

$x = 15, 75, 195, 255$

$\boxed{2}$

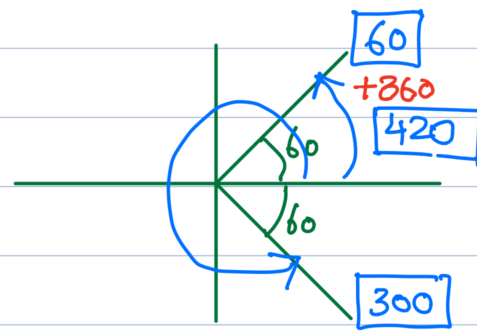
$$\cos(x+70) = \frac{1}{2} \qquad 0 < x < 360$$

$$\underline{\quad +70 \quad}$$

$$A = x+70 \qquad 70 < x+70 < 430$$

$$\cos A = \frac{1}{2} \qquad 70 < A < 430$$

$$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$



out of range

$$A = \cancel{60},\ 300,\ 420$$

$$x+70 = 300,\ 420$$

$$x = 230,\ 350$$

$\boxed{3}$

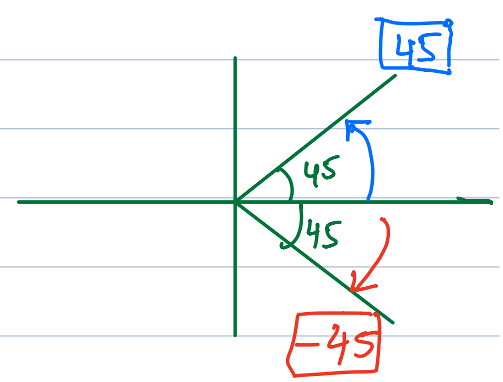
$$\cos(x-80) = \frac{1}{\sqrt{2}} \qquad 0 < x < 360$$

$$\underline{\quad -80 \quad}$$

$$A = x-80 \qquad -80 < x-80 < 280$$

$$\cos A = \frac{1}{\sqrt{2}} \qquad -80 < A < 280$$

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45$$



$$A = -45,\ 45$$

$$x-80 = -45,\ 45$$

$$x = 35,\ 125.$$

$\boxed{4}$ $\sin(2x+40) = \frac{1}{2}$

$0 < x < 180$

$\times 2$

$0 < 2x < 360$

$+40$

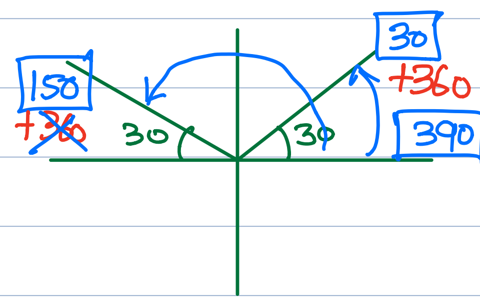
$A = 2x+40$

$40 < 2x+40 < 400$

$\sin A = \frac{1}{2}$

$40 < A < 400$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = 30$



$A = \cancel{30}, 150, 390$

$2x + 40 = 150, 390$

$2x = 110, 350$

$\boxed{x = 55, 175}$

IDENTITIES: (17)

RECIPROCAL

$$\frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\frac{1}{\tan\theta} = \cot\theta$$

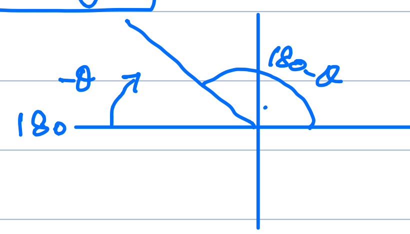
$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\frac{\cos\theta}{\sin\theta} = \cot\theta$$

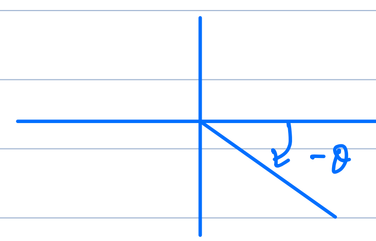
$$\sin^2\theta + \cos^2\theta \equiv 1$$
$$1 + \tan^2\theta \equiv \sec^2\theta$$
$$1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$$

$\boxed{\theta = \text{Acute angle}}$

$$\sin(180-\theta) \equiv \sin\theta$$
$$\cos(180-\theta) \equiv -\cos\theta$$
$$\tan(180-\theta) \equiv -\tan\theta$$

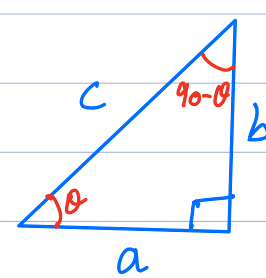


$$\sin(-\theta) \equiv -\sin\theta$$
$$\cos(-\theta) \equiv \cos\theta$$
$$\tan(-\theta) \equiv -\tan\theta$$



M1, P1

$$\sin(90-\theta) \equiv \cos\theta$$
$$\cos(90-\theta) \equiv \sin\theta$$
$$\tan(90-\theta) \equiv \frac{1}{\tan\theta}$$



Symbols:

$$\sin^2\theta = (\sin\theta)^2$$

$$\sin x^2 = \sin(x^2)$$

$$\sin^2 30 \longrightarrow (\sin 30)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\sin 30^2 \longrightarrow \sin(30^2) = \sin(900) = \boxed{}$$

Ex

$$2\cos(180-\theta) = 1$$

$$2(-\cos\theta) = 1$$

$$-2\cos\theta = 1$$

$$\cos\theta = \frac{-1}{2}$$

Q

$$2\cos(-\theta) = 1$$

$$\cos(-\theta) = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$

PROVING IDENTITIES

1) YOU ARE ALLOWED TO WORK WITH ONLY ONESIDE AND PROVE IT EQUAL TO OTHER SIDE.

2) YOU ARE NOT ALLOWED TO SIMPLIFY BOTH SIDES AT SAMETIME.

$\boxed{\text{PROOF}}$

$\boxed{\text{SQUARE}}$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$a^2 - b^2 = (a+b)(a-b)$

$\boxed{\text{NO SQUARES}}$

BRING EVERYTHING TO $\boxed{\text{SIN}}$ & $\boxed{\text{COS}}$ only and SIMPLIFY.

32 (i) Prove the identity $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$. [2]

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x}$$ (shown) (Q.E.D).

13 Prove the identity

$$\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}\equiv\frac{2}{\cos x}.$$ [4]

$$\frac{(1+\sin x)(1+\sin x)+(\cos x)(\cos x)}{\cos x\,(1+\sin x)}$$

$$\frac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x\,(1+\sin x)}$$

$$\frac{1+2\sin x+1}{\cos x\,(1+\sin x)}$$

$$\frac{2+2\sin x}{\cos x(1+\sin x)}$$

$$\frac{2\cancel{(1+\sin x)}}{\cos x\,\cancel{(1+\sin x)}}=\frac{2}{\cos x}$$ (Proven)

38 (i) Show that $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}\equiv\dfrac{1}{\sin^2\theta-\cos^2\theta}$. [3]

$$\frac{\sin\theta(\sin\theta-\cos\theta)+\cos\theta(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$$

$$\frac{\sin^2\theta-\cancel{\sin\theta\cos\theta}+\cancel{\sin\theta\cos\theta}+\cos^2\theta}{(\sin\theta)^2-(\cos\theta)^2}$$

$$\frac{1}{\sin^2\theta-\cos^2\theta}$$

27 (i) Prove the identity $\left(\dfrac{1}{\sin\theta}-\dfrac{1}{\tan\theta}\right)^2\equiv\dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

$$\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$$

$$\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$\frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta=1-\cos^2\theta.$

$$\frac{(1-\cos\theta)^2}{1^2-\cos^2\theta}$$

$$\frac{(1-\cos\theta)^{\cancel{2}}}{\cancel{(1-\cos\theta)}(1+\cos\theta)}=\frac{1-\cos\theta}{1+\cos\theta}.$$

21 (i) Prove the identity $\frac{\sin x \tan x}{1 - \cos x} \equiv 1 + \frac{1}{\cos x}$.

$$\frac{\sin x \cdot \frac{\sin x}{\cos x}}{1 - \cos x}$$

$$\sin x \cdot \frac{\sin x}{\cos x} \div (1 - \cos x)$$

$$\frac{\sin^2 x}{\cos x} \times \frac{1}{(1 - \cos x)}$$

$$\frac{\sin^2 x}{\cos x\,(1 - \cos x)}$$

$$\frac{1^2 - \cos^2 x}{\cos x\,(1 - \cos x)}$$

$$\frac{(1 + \cos x)\cancel{(1 - \cos x)}}{\cos x\,\cancel{(1 - \cos x)}}$$

$$\frac{1 + \cos x}{\cos x}$$

$$\frac{1}{\cos x} + \frac{\cos x}{\cos x}$$

$$\boxed{\frac{1}{\cos x} + 1}$$ Shown.

CORRECT

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

WRONG.

$$\frac{c}{a+b} = \frac{c}{a} + \frac{c}{b} \quad \times \quad \times$$

DONE!

ZAINEMATICS

FOR THE LOVE OF MATHS

Differentiation

P1

Compiled by Rafay Mushtaq

zainematics zainematics zainematics

TRIG → 5 types

FUNCTIONS 5 TYPES

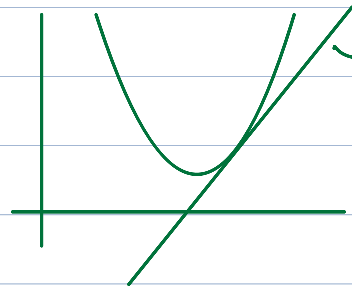
DIFF 3 Types

Integ 3 Types.

DIFFERENTIATION :-

GRADIENT OF TANGENT (CURVE)

OLevels:



→ (gradient of tangent)

$\frac{y_2 - y_1}{x_2 - x_1}$

$\frac{\text{change in } y}{\text{change in } x}$

delta ← $\frac{\Delta y}{\Delta x}$

$\boxed{\text{SYMBOLS}}$

$y \xrightarrow{\text{diff}} \frac{dy}{dx}$

$y \xrightarrow{\text{diff}} y'$

HOW TO DIFF :-

BASIC RULES :-

$\boxed{1}$

$x \longrightarrow 1$

$2x \longrightarrow 2$

$10x \longrightarrow 10$

$12x \longrightarrow 12$

$\boxed{2}$ Alone constant $\longrightarrow 0$

$6 \longrightarrow 0$

$4 \longrightarrow 0$

$100 \longrightarrow 0$

eg $y = 5 - 3x$

$\frac{dy}{dx} = 0 - 3 = -3$

POWER RULE :- (THREE STEP PROCESS)

$$(\square)^n \longrightarrow \overset{1}{n}\,(\square)^{\overset{2}{n-1}} \overset{3}{\times \square'}$$

1

$$y = \boxed{x}^7$$

$$\frac{dy}{dx} = 7x^6 \times 1$$

$$\frac{dy}{dx} = 7x^6$$

2

$$y = 12\boxed{x}^3$$

$$\frac{dy}{dx} = 36x^2 \times 1$$

$$\frac{dy}{dx} = 36x^2$$

3

$$y = (\boxed{12x})^3$$

$$\frac{dy}{dx} = 3(12x)^2 \times 12$$

$$\frac{dy}{dx} = 36(12x)^2$$

4

$$y = 9\boxed{x}^3 - 12x + 7$$

$$\frac{dy}{dx} = 27x^2(1) - 12 + 0$$

$$\frac{dy}{dx} = 27x^2 - 12$$

5

$$y = (\boxed{3x-1})^5$$

$$\frac{dy}{dx} = 5(3x-1)^4 \times (3-0)$$

$$\frac{dy}{dx} = 15(3x-1)^4$$

6

$$y = 6\boxed{x}^2 - 9x + 8$$

$$\frac{dy}{dx} = 12x^1(1) - 9 + 0$$

$$\frac{dy}{dx} = 12x - 9$$

7

$$y = (\boxed{2x+5})^3 - 9\boxed{x}^2$$

$$\frac{dy}{dx} = 3(2x+5)^2(2+0) - 18x^1(1)$$

$$\frac{dy}{dx} = 6(2x+5)^2 - 18x$$

YOU CANNOT DIFFERENTIATE IN DENOMINATOR!

8

$$y = 3x^2 - \frac{2}{x^3}$$

$$y = 3\boxed{x}^2 - 2\boxed{x}^{-3}$$

$$\frac{dy}{dx} = 6x^1(1) + 6x^{-4}(1)$$

$$\frac{dy}{dx} = 6x + \frac{6}{x^4}$$

9

$$y = \frac{12}{(2x-3)^2}$$

$$y = 12\left(\boxed{2x-3}\right)^{-2}$$

$$\frac{dy}{dx} = -24(2x-3)^{-3}(2-0)$$

$$\frac{dy}{dx} = \frac{-48}{(2x-3)^3}$$

9

$$y = \frac{9x^2+2}{x}$$

$$y = \frac{9x^2}{x} + \frac{2}{x}$$

$$y = 9x + \frac{2}{x}$$

$$y = 9x + 2\boxed{x}^{-1}$$

$$\frac{dy}{dx} = 9 + (-2)x^{-2}(1)$$

$$\boxed{\frac{dy}{dx} = 9 - \frac{2}{x^2}}$$

OUTCOMES :-

$\frac{dy}{dx}$ = gradient of TANGENT (curve).

Q $y = x^2 - 2x + 4$

(i) Find expression for $\frac{dy}{dx}$

$$y = \boxed{x}^2 - 2x + 4$$

$$\frac{dy}{dx} = 2x^1(1) - 2 + 0 = \boxed{2x - 2}$$

(ii) Find gradient of tangent at $x = 2$

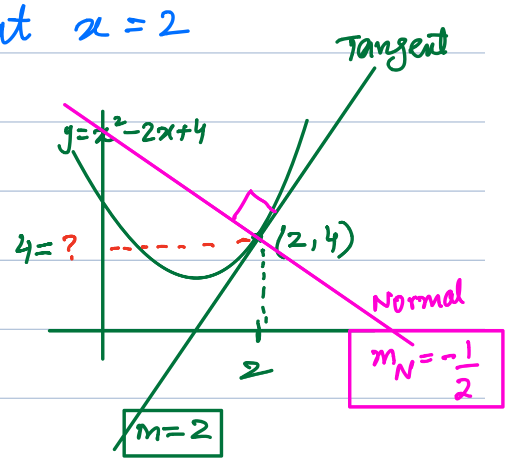
$$\frac{dy}{dx} = 2x - 2 \qquad x = 2$$

$$= 2(2) - 2$$

gradient of tangent $= \boxed{2 = m_T}$

(iii) Find equation of tangent at $x = 2$

For y coordinate put $x = 2$ in curve equation.



$$y = 2^2 - 2(2) + 4 = 4$$

Point $(2, 4)$, $m_T = 2$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - 2)$$

$$y - 4 = 2x - 4$$

$\boxed{y = 2x}$ TANGENT.

(iv) Find equation of normal to curve at $x = 2$.

$m_T = 2 \longrightarrow m_N = -\frac{1}{2}$

$y - 4 = -\frac{1}{2}(x - 2)$

Point $(2, 4)$

$2y - 8 = -x + 2$

$\boxed{2y = -x + 10}$

You cannot tell gradient of normal directly.

$y \xrightarrow{\text{diff}} \frac{dy}{dx} \xrightarrow[\text{reciprocal}]{\text{Negative}} m_N$

$\frac{dy}{dx} \downarrow m_T$

2 Find the gradient of the curve $y = \dfrac{12}{x^2 - 4x}$ at the point where $x = 3$. [4]

$\boxed{\frac{dy}{dx}}$

$y = 12\left(\boxed{x^2 - 4x}\right)^{-1}$

$\sqrt{x^2}$

$2x^1(1)$

$\frac{dy}{dx} = -12(x^2 - 4x)^{-2} \times (2x - 4)$

$\frac{dy}{dx} = \frac{-12(2x - 4)}{(x^2 - 4x)^2}$ $\quad x = 3$

$= \frac{-12(2(3) - 4)}{(3^2 - 4(3))^2} = -\frac{8}{3}$

22 A curve has equation $y = \dfrac{4}{3x-4}$ and $P\,(2,\ 2)$ is a point on the curve.

(i) Find the equation of the tangent to the curve at P. [4]

(ii) Find the angle that this tangent makes with the x-axis. [2]

(i) $y = 4\,(\boxed{3x-4})^{-1}$

$\dfrac{dy}{dx} = -4(3x-4)^{-2}(3)$

$\boxed{\dfrac{dy}{dx} = \dfrac{-12}{(3x-4)^2}}$ $\quad x = 2$

$m_T = \dfrac{-12}{(3(2)-4)^2} = \dfrac{-12}{4} = -3$

$m_T = -3$, $P(2,2)$

$y - 2 = -3(x-2)$

$y = -3x+6+2$

$\boxed{y = -3x+8}$

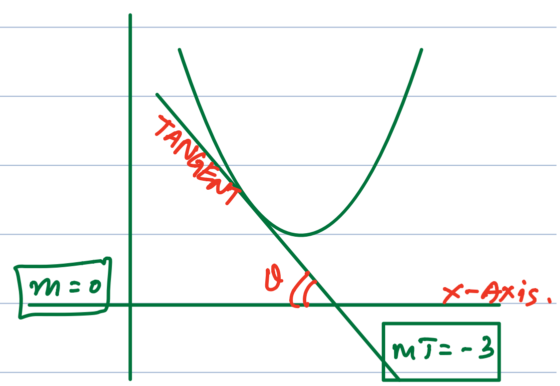
(ii) $\tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

$\tan\theta = \dfrac{0-(-3)}{1+(0)(-3)}$

$\tan\theta = 3$

$\theta = \tan^{-1}(3)$

$\theta = 71.565$



Please wait. I'm about to reach academy. In traffic hn.

11 The equation of a curve is $y = \dfrac{12}{x^2+3}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

(ii) Find the equation of the normal to the curve at the point $P\,(1, 3)$. [3]

$y = 12(x^2+3)^{-1}$

$\dfrac{dy}{dx} = -12(x^2+3)^{-2}(2x)$

$\dfrac{dy}{dx} = \dfrac{-24x}{(x^2+3)^2}$

$m_T = \dfrac{-24(1)}{(1^2+3)^2} = \dfrac{-24}{16} = \dfrac{-3}{2}$

$m_N = \dfrac{2}{3}$, $P(1,3)$

$y-3 = \dfrac{2}{3}(x-1)$

$3y-9 = 2x-2$

$\boxed{3y = 2x+7}$

10 The equation of a curve is $y = 5 - \dfrac{8}{x}$

(i) Show that the equation of the normal to the curve at the point $P\,(2, 1)$ is $2y + x = 4$. [4] Diff

point of intersection (simultaneously).

This normal meets the curve again at the point Q.

(ii) Find the coordinates of Q. coordinate geo. [3]

(iii) Find the length of PQ. [2]

$y = 5 - 8x^{-1}$

$\dfrac{dy}{dx} = 0 - (-8)x^{-2}(1) = \dfrac{8}{x^2}$

$m_T = \dfrac{8}{2^2} = \dfrac{8}{4} = 2$

$m_N = -\dfrac{1}{2}$ $P(2,1)$

$y - 1 = -\dfrac{1}{2}(x-2)$

$2y-2 = -x+2$

$2y = -x+4$

$\boxed{2y + x = 4}$

$\boxed{\text{ii}}$ $\boxed{Q}$ $y = 5 - \dfrac{8}{x}$ $2y + x = 4$

$$5 - \frac{8}{x} = \frac{4 - x}{2}$$

$$\frac{5x - 8}{x} = \frac{4 - x}{2}$$

$$10x - 16 = 4x - x^2$$

$$x^2 + 6x - 16 = 0$$

$$x^2 + 8x - 2x - 16 = 0$$

$$x(x+8) - 2(x+8) = 0$$

$$(x-2)(x+8) = 0$$

$x = 2$, $x = -8$

<u>PoinP</u> <u>Point Q</u>

$$y = \frac{4 - (-8)}{2}$$

$$y = 6$$

$$Q(-8, 6)$$

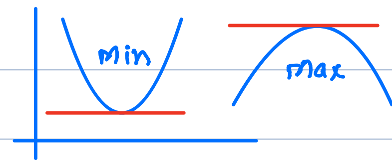
$P(2,1)$ $Q(-8,6)$

Length PQ $= \sqrt{(-8-2)^2 + (6-1)^2}$

$= \sqrt{125}$

$=$ ___

STATIONARY POINT
TURNING POINT
VERTEX
MAXIMUM POINT
MINIMUM POINT
CRITICAL POINT (SAT)



TANGENT = HORIZONTAL = Gradient is zero.

GRADIENT OF TANGENT IS ZERO

$$\boxed{\frac{dy}{dx} = 0}$$

Q. $y = x^2 + 12x - 3$

Find x-coordinates of its stationary point.

$$\frac{dy}{dx} = 2x + 12$$

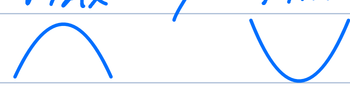
$$0 = 2x + 12$$

$$2x = -12$$

$$x = -6$$

NATURE OF A STATIONARY POINT

MAX / MIN



$$y \xrightarrow{\text{diff}} \frac{dy}{dx} \xrightarrow{\text{diff}} \frac{d^2y}{dx^2} \begin{cases} \boxed{+ve} & \text{MIN} \\ \boxed{-ve} & \text{MAX.} \end{cases}$$

$$y \xrightarrow{\text{diff}} y' \xrightarrow{\text{diff}} y''$$

7 The equation of a curve is $y = (2x - 3)^3 - 6x$.

(i) Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x. [3]

(ii) Find the x-coordinates of the two stationary points and determine the nature of each stationary point. [5]

(i) $y = (\boxed{2x-3})^3 - 6x$

$\frac{dy}{dx} = 3(2x-3)^2(2) - 6$

$\boxed{\frac{dy}{dx} = 6(\boxed{2x-3})^2 - 6}$

$\frac{d^2y}{dx^2} = 12(2x-3)^1(2) - 0$

$\boxed{\frac{d^2y}{dx^2} = 24(2x-3)}$

(ii) STATIONARY POINTS

$\frac{dy}{dx} = 0$

$6(2x-3)^2 - 6 = 0$

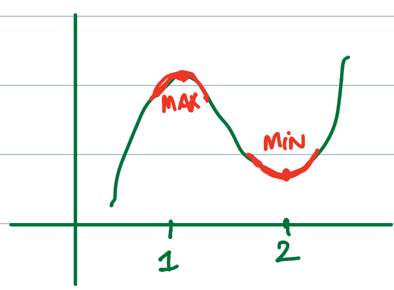
$6(2x-3)^2 = 6$

$\sqrt{(2x-3)^2} = \pm\sqrt{1}$

$2x - 3 = \pm 1$

$x = \frac{3 \pm 1}{2}$

$\boxed{x = 2}$, $\boxed{x = 1}$



NATURE :-

$\frac{d^2y}{dx^2} = 24(2x-3)$

$\boxed{x=2}$ $\frac{d^2y}{dx^2} = 24(2(2)-3) = 24\,(+)$ (MIN)

$\boxed{x=1}$ $\frac{d^2y}{dx^2} = 24(2(1)-3) = -24\,(-)$ (max)

$$y \xrightarrow{\text{diff}} \frac{dy}{dx}. \xrightarrow{\text{diff}} \frac{d^2y}{dx^2}.$$

+ve Min

-ve Max.

$$\boxed{\frac{dy}{dx}=0}$$

STATIONARY POINTS

52 A curve has equation $y = \frac{8}{x} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5]

(i) $y = 8x^{-1} + 2x$

$\frac{dy}{dx} = -8x^{-2}(1) + 2$

$$\boxed{\frac{dy}{dx} = -8x^{-2} + 2}$$

$\frac{d^2y}{dx^2} = 16x^{-3}(1)$

$$\boxed{\frac{d^2y}{dx^2} = \frac{16}{x^3}}$$

(ii) STATIONARY POINT :-

$\frac{dy}{dx} = 0$

$-8x^{-2} + 2 = 0$

$-\frac{8}{x^2} + 2 = 0$

$2 = \frac{8}{x^2}$

$2x^2 = 8$

$x^2 = 4$

$$\boxed{x = 2, -2}$$

NATURE

$\boxed{x=2}$ $\frac{d^2y}{dx^2} = \frac{16}{2^3} = 2$

+ Min

$\boxed{x=-2}$ $\frac{d^2y}{dx^2} = \frac{16}{(-2)^3}$

$= -2$

Max

20 A curve has equation $y = \dfrac{1}{x-3} + x$.

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [2]

(ii) Find the coordinates of the maximum point A and the minimum point B on the curve. [5]

$y = (x-3)^{-1} + x$

$\dfrac{dy}{dx} = -1\,(x-3)^{-2}(1) + 1$

$$\boxed{\dfrac{dy}{dx} = -1\,(x-3)^{-2} + 1}$$

$\dfrac{d^2y}{dx^2} = 2\,(x-3)^{-3}(1) + 0$

$$\boxed{\dfrac{d^2y}{dx^2} = 2\,(x-3)^{-3}}$$

MAX / MIN POINT (TURNING POINT).

$0 = -1(x-3)^{-2} + 1$

$0 = \dfrac{-1}{(x-3)^2} + 1$

$\dfrac{1}{(x-3)^2} = 1$

$(x-3)^2 = 1$

$x - 3 = \pm 1$

$x = 3 \pm 1$

$x = 4$,	$x = 2$
$y = \dfrac{1}{4-3} + 4$	$y = \dfrac{1}{2-3} + 2$
$y = 5$	$y = 1$
$B(4,5)$	$A(2,1)$

$x = 4$, $\dfrac{d^2y}{dx^2} = \dfrac{2}{(4-3)^3} = 2$ (+ve) Min B

$x = 2$, $\dfrac{d^2y}{dx^2} = \dfrac{2}{(2-3)^3} = -2$ (−ve) Max A

30 The curve $y = \dfrac{10}{2x+1} - 2$ intersects the x-axis at A. The tangent to the curve at A intersects the y-axis at C.

(i) Show that the equation of AC is $5y + 4x = 8$. [5]

(ii) Find the distance AC. [2]

$\boxed{A}$ x-axis, $y=0$, $0 = \dfrac{10}{2x+1} - 2$

$$2 = \frac{10}{2x+1}$$

$$4x+2 = 10$$

$A = (2,0)$ $\boxed{x=2}$

TANGENT AT A:

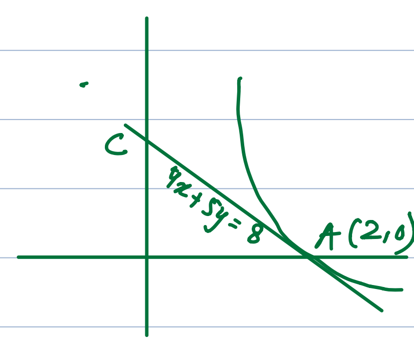
$$y = 10(2x+1)^{-1} - 2$$

$$\frac{dy}{dx} = 10(-1)(2x+1)^{-2}(2) - 0$$

$$\frac{dy}{dx} = \frac{-20}{(2x+1)^2} \qquad x=2$$

$$m_T = \frac{-20}{(2(2)+1)^2} = -\frac{4}{5}$$

$m = -\frac{4}{5}$ $A\ (2,0)$



$$y - 0 = -\frac{4}{5}(x-2)$$

$\boxed{5y = -4x + 8}$ Tangent.

$4x + 5y = 8$

$\boxed{C}$ y-intercept: $x=0$

$4(0)+5y=8$

$y=1.6$

$A(2,0)$ $\quad C(0, 1.6)$

$AC = \sqrt{(0-2)^2+(1.6-0)^2} = \boxed{}$

12

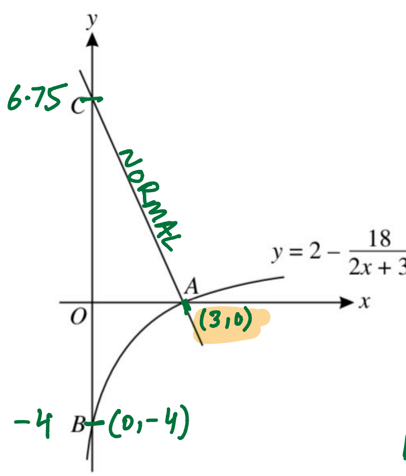
$\boxed{A}$ x-axis. $y=0$

$0 = 2 - \dfrac{18}{2x+3}$

$\dfrac{18}{2x+3} = 2$

$18 = 4x+6$

$\boxed{x=3}$



$\boxed{B}$ $x=0$, y-axis

$y = 2 - \dfrac{18}{2(0)+3}$

$y=-4$

$\boxed{C}$ $x=0$, y axis

$9(0)+4y=27$

$4y=27$

$y=6.75$

$BC = 6.75+4 = \boxed{10.75}$

The diagram shows part of the curve $y = 2 - \dfrac{18}{2x+3}$, which crosses the x-axis at A and the y-axis at B. The normal to the curve at A crosses the y-axis at C.

(i) Show that the equation of the line AC is $9x + 4y = 27$. [6]

Normal.

(ii) Find the length of BC. [2]

$y = 2 - 18(2x+3)^{-1}$

$\dfrac{dy}{dx} = 0 - 18(-1)(2x+3)^{-2}(2)$

$\dfrac{dy}{dx} = \dfrac{36}{(2x+3)^2}$ $\qquad x = 3$

$m_T = \dfrac{36}{(2(3)+3)^2} = \dfrac{4}{9}$

NORMAL:

$m = -\dfrac{9}{4}$, $A(3,0)$

$y - 0 = -\dfrac{9}{4}(x-3)$

$4y = -9x+27$

$\boxed{9x+4y=27}$ $\boxed{AC}$

5 A curve has equation $y = \dfrac{k}{x}$. Given that the $\underbrace{\text{gradient of the curve}}_{\frac{dy}{dx}=-3}$ is -3 when $x = 2$, find the value of the constant k. [3]

$\frac{dy}{dx} = -3 \quad , \quad x = 2$

$y = k x^{-1}$

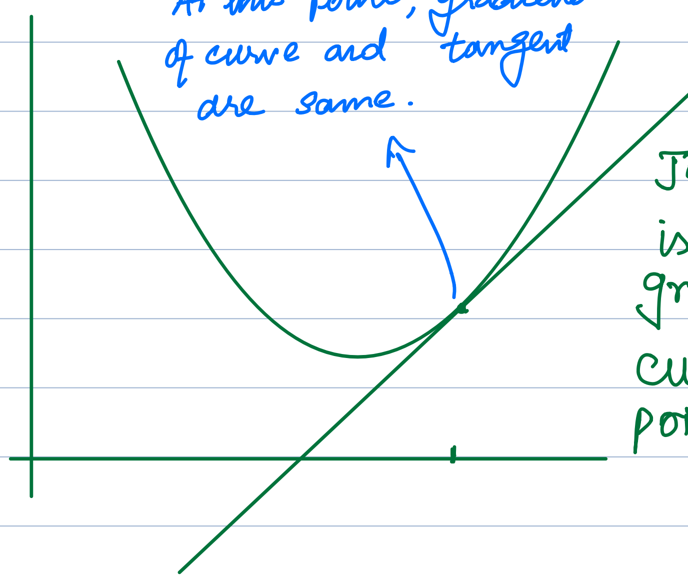
$\frac{dy}{dx} = k(-1)x^{-2}(1)$

$\frac{dy}{dx} = \frac{-k}{x^2}$

$-3 = \frac{-k}{(2)^2}$

$k = 12$

At this point, gradient of curve and tangent are same.



Job of Tangent is to tell gradient of curve at that point.

(eg) $y = 3\sqrt{x} + 2x \longrightarrow y = 3\sqrt{x} + 2x$

(eg) $y = 3\sqrt{(x+5)} \longrightarrow y = 3\sqrt{x+5}$

$$y = \sqrt{2x+3}$$

$$y = (2x+3)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{\cancel{2}}(2x+3)^{-\frac{1}{2}} \times (\cancel{2})$$

$$\frac{dy}{dx} = \frac{1}{(2x+3)^{\frac{1}{2}}} = \frac{1}{\sqrt{2x+3}}$$

RATE OF CHANGE

Rate of change of $x = \frac{dx}{dt}$

Rate of change of volume $= \frac{dV}{dt}$

Rate of change of Area $= \frac{dA}{dt}$

LAYOUT

Two main variables + Connecting equation

$y \qquad x \qquad y = 2x^2 + 9x - 1$

$$\frac{dy}{dt} \qquad \frac{dx}{dt} \qquad \frac{dy}{dx}$$

CHAIN RULE: $$\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$$

Q. A circular water pond is expanding such that its radius is increasing at rate of 4 m/s. Find rate of increase of its area when $r = 15$.

connecting equation.

$r \qquad A \qquad A = \pi r^2$

$$\frac{dr}{dt} \qquad \frac{dA}{dt} \qquad \frac{dA}{dr}$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$A = \pi \boxed{r}^2$

$$\frac{dA}{dr} = 2\pi r^1(1)$$

$$\boxed{\frac{dA}{dr} = 2\pi r}$$

Data in Question

$\frac{dr}{dt} = 4$

$\frac{dA}{dt} = ?$

$r = 15$

$$\frac{dA}{dt} = 2\pi r \times 4$$

$$\frac{dA}{dt} = 2\pi(15) \times 4$$

$$\frac{dA}{dt} = 120\pi = \boxed{}$$

21 The volume of a spherical balloon is increasing at a constant rate of 50 cm^3 per second. Find the rate of increase of the radius when the radius is 10 cm. [Volume of a sphere = $\frac{4}{3}\pi r^3$.] [4]

MAIN	VARIABLES	CONN. EQ.		Data in question
r	V	$V = \frac{4}{3}\pi r^3$	$V = \frac{4}{3}\pi \boxed{r}^{3}$	$\frac{dV}{dt} = 50$
$\frac{dr}{dt}$	$\frac{dV}{dt}$	$\frac{dV}{dr}$	$\frac{dV}{dr} = \frac{4}{3}\pi(3)r^2(1)$	$\frac{dr}{dt} = ?$ $r = 10$
			$\frac{dV}{dr} = 4\pi r^2$	

$$\frac{dV}{dt} = \frac{dr}{dt} \times \frac{dV}{dr}$$

$$50 = \frac{dr}{dt} \times 4\pi r^2$$

$$50 = \frac{dr}{dt} \times 4\pi(10)^2$$

$$\frac{dr}{dt} = \frac{50}{400\pi} = \boxed{}$$

31 An oil pipeline under the sea is leaking oil and a circular patch of oil has formed on the surface of the sea. At midday the radius of the patch of oil is 50 m and is increasing at a rate of 3 metres per hour. Find the rate at which the area of the oil is increasing at midday. [4]

$A \quad r \quad A = \pi r^2$

$\frac{dA}{dt} = \frac{dr}{dt} \times \frac{dA}{dr}$

$\frac{dA}{dt} = 3 \times 2\pi r$

$= 3 \times 2\pi(50)$

$= 300\pi$

$A = \pi \boxed{r}^2$

$\frac{dA}{dr} = 2\pi r^1(1)$

$\frac{dA}{dr} = 2\pi r$

Data

$r = 50$

$\frac{dr}{dt} = 3$

$\frac{dA}{dt} = ?$

11 The equation of a curve is $y = \dfrac{12}{x^2 + 3}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

point $P(1, 3)$.

(iii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.012 units per second. Find the rate of change of the y-coordinate as the point passes through P. $(x=1, y=3)$ [2]

(i)

$y = 12(x^2+3)^{-1}$

$\frac{dy}{dx} = -12(x^2+3)^{-2}(2x)$

$\frac{dy}{dx} = \frac{-24x}{(x^2+3)^2}$

(iii)

$x \quad y \quad y = \frac{12}{x^2+3}$

$\frac{dx}{dt} \quad \frac{dy}{dt} \quad \frac{dy}{dx}$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-24x}{(x^2+3)^2} \times 0.012$

at Point P $\boxed{x=1}$

$\frac{dy}{dt} = \frac{-24(1)}{(1^2+3)^2} \times 0.012 = \boxed{-0.018}$

SCENARIO BASED

ALWAYS ATTEMPT THE DIFFERENTIATION PART FIRST.

Question Layout

Substitution

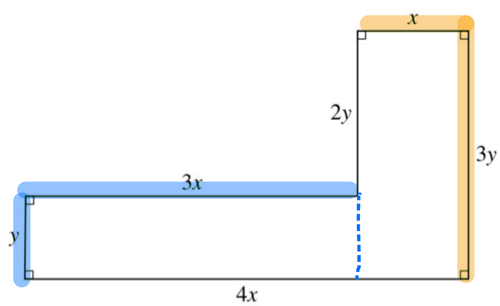
PROOF

START

Proof gets stuck after 1 to 2 steps

Solves

26



The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48 m.

(i) Find an expression for y in terms of x. [1]

(ii) Given that the area of the garden is A m^2, show that $A = 48x - 8x^2$. [2]

(iii) Given that x can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

(i)

$$P = y + 2y + 3y + 4x + 3x + x$$

$$P = 6y + 8x$$

$$48 = 6y + 8x$$

$$\boxed{y = \frac{48 - 8x}{6}}$$

(ii)

$$A = (y)(3x) + (x)(3y)$$

$$A = 3xy + 3xy$$

$$A = 6xy$$

$$A = \cancel{6}x\left(\frac{48 - 8x}{\cancel{6}}\right)$$

$$\boxed{A = 48x - 8x^2}$$

(iii) $A = 48x - 8x^2$

Max
$\downarrow$
Stationary Point.
$\downarrow$
diff $= 0$

$$\boxed{\frac{dA}{dx} = 48 - 16x}$$

$$0 = 48 - 16x$$
$$16x = 48$$
$$x = 3$$

$A = 48(3) - 8(3)^2 = 72$ (Max. value of area).

$$\frac{dA}{dx} = 48 - 16x$$

$$\frac{d^2A}{dx^2} = -16 \quad (\text{Max}).$$

13 A solid rectangular block has a square base of side x cm. The height of the block is h cm and the total surface area of the block is $96\,\text{cm}^2$.

(i) Express h in terms of x and show that the volume, $V\,\text{cm}^3$, of the block is given by

$$V = 24x - \tfrac{1}{2}x^3.$$ [3]

Given that x can vary,

(ii) find the stationary value of V, [3]

(iii) determine whether this stationary value is a maximum or a minimum. [2]

Total SA $= 96$

$$2(x^2) + 4(x)(h) = 96$$

$$2x^2 + 4xh = 96$$

$$\boxed{h = \frac{96 - 2x^2}{4x}}$$

$$V = (x)(x)(h)$$

$$V = x^2 h$$

$$V = x^{\cancel{2}}\left(\frac{96 - 2x^2}{4\cancel{x}}\right)$$

$$V = \frac{96x - 2x^3}{4}$$

$$V = \frac{96x}{4} - \frac{2x^3}{4}$$

$$\boxed{V = 24x - \frac{1}{2}x^3}$$

(ii) $$V = 24x - \frac{1}{2}x^3$$

$$\frac{dV}{dx} = 24 - \frac{1}{2}(3)(x)^2(1)$$

$$\boxed{\frac{dV}{dx} = 24 - \frac{3}{2}x^2}$$

$$0 = 24 - \frac{3}{2}x^2$$

$$\frac{3}{2}x^2 = 24$$

$$x^2 = 16$$

$$x = 4$$

$$V = 24(4) - \frac{1}{2}(4)^3$$

$$V = 64$$

(iii) $$\frac{dV}{dx} = 24 - \frac{3}{2}x^2$$

$$\frac{d^2V}{dx^2} = 0 - \frac{3}{2}(2x)$$

$$\frac{d^2V}{dx^2} = -3x \qquad \boxed{x = 4}$$

$$\frac{d^2V}{dx^2} = -3(4) = -12 \quad (\text{Max})$$

DIFFERENTIATION

QUESTIONS:

TYPE 1	EQUATION OF TANGENT AND NORMAL.
TYPE 2	STATIONARY POINTS AND ITS NATURE.
TYPE 3	SCENARIO BASED PROOF QUESTION
TYPE 4	RATE OF CHANGE.
TYPE 5	INCREASING / DECREASING FUNCTIONS.

ZAINEMATICS

FOR THE LOVE OF MATHS

Integration

P1

Compiled by Rafay Mushtaq

INTEGRATION

OUTCOMES : 1) AREA UNDER GRAPH

2) VOLUME OF ROTATION

3) Reverse working.

$$
\begin{tikzcd}
y \arrow[r, bend left, "\text{diff}"] & \frac{dy}{dx} \arrow[r, bend left, "\text{diff}"] \arrow[l, bend left, "\text{integration}"] & \frac{d^2y}{dx^2} \arrow[l, bend left, "\text{integration}"]
\end{tikzcd}
$$

SYMBOL

$$\int \bigcirc \, dx$$

BASE RULE :-

$1 \longrightarrow x$

alone constant $(K) \longrightarrow Kx$

$2 \longrightarrow 2x$

$5 \longrightarrow 5x$

POWER RULE :

$$(\square)^{n} \longrightarrow \frac{(\square)^{n+1}}{n+1}$$

RULES:

1) YOU ARE NOT ALLOWED TO INTEGRATE POWER UNLESS "DIFFERENTIATION OF BOX" $\square'$ IS PRESENT OUTSIDE OPERATOR.

2) ONCE THIS CONDITION IS FULFILLED, THREE THINGS WILL DISAPPEAR,

$$\int \square' \qquad dx$$

AND POWER IS ALLOWED TO BE INTEGRATED.

EXAMPLE:

1) $\int x^8 \, dx$

$$\int 1 \, \boxed{x}^{8} \, dx \qquad \square = x \qquad \square' = 1$$

$$\frac{x^9}{9}$$

2) $\int (x+5)^7 \, dx$

$\int 1 (\boxed{x+5})^{7} \, dx$ $\quad \square = x+5$

$\square' = 1$

$\frac{(x+5)^8}{8}$

3) $\int (2x+4)^6 \, dx$

$\frac{1}{2} \int 2 (\boxed{2x+4})^{6} \, dx$ $\quad \square = 2x+4$

$\square' = 2$

$\frac{1}{2} \times \frac{(2x+4)^7}{7}$

$\frac{(2x+4)^7}{14}$

3) $$\int (5x+8)^7\,dx$$

$$\frac{1}{5}\int 5\left(\boxed{5x+8}\right)^{\underline{\underline{7}}}\,dx$$

$\square = 5x+8$

$\square' = 5$

$$\frac{1}{5}\times\frac{(5x+8)^8}{8}$$

$$\frac{(5x+8)^8}{40}$$

RULES:

1) YOU ARE NOT ALLOWED TO INTEGRATE POWER UNLESS "DIFFERENTIATION OF BOX" $\square'$ IS PRESENT OUTSIDE OPERATOR.

2) ONCE THIS CONDITION IS FULFILLED, THREE THINGS WILL DISAPPEAR,

$$\int \square' \qquad dx$$

AND POWER IS ALLOWED TO BE INTEGRATED.

4) $$\int x(x^2+5)^3\,dx$$

$$\frac{1}{2}\int 2x\left(\boxed{x^2+5}\right)^{\underline{\underline{3}}}\,dx$$

$\square = x^2+5$

$\square' = 2x$

$$\frac{1}{2}\times\frac{(x^2+5)^4}{4}$$

$$\frac{(x^2+5)^4}{8}$$

5) $$\int x^2 (9x^3+3)^4 \, dx$$

$$\frac{1}{27}\int 27x^2 \left(\boxed{9x^3+3}\right)^{\underline{4}} dx$$

$\square = 9x^3+3$

$\square' = 27x^2$

$$\frac{1}{27} \times \frac{(9x^3+3)^5}{5}$$

$\boxed{6}$ $$\int (2x+3)^7 \, dx$$

$$\frac{1}{2}\int 2\left(\boxed{2x+3}\right)^{\underline{7}} dx$$

$\square = 2x+3$

$\square' = 2$

$$\frac{1}{2} \times \frac{(2x+3)^8}{8}$$

THERE ARE ONLY 2 TYPES OF INTEGRATION THAT YOU WILL FACE IN P1.

$(\square)^n$

In this case, you have to be careful about conditions of integration.

There is no whole power. Power is on each x term individually

↓

you can integrate without checking conditions.

eg 1) $\int 4x^7\,dx$

No whole power.
Power is directly on x-term.
You can integrate directly.

$\int 4\boxed{x}^{\underline{\underline{7}}}\,dx$

$4\dfrac{x^8}{8}$

2) $\int (x^3 + 2x^2 + 5)\,dx$

No whole power
Treat as seperate terms.

$\dfrac{x^4}{4} + 2\dfrac{x^3}{3} + 5x$

Q $\int x(3x^2+9)^5\,dx$

$\dfrac{1}{6}\int 6x(\boxed{3x^2+9})^{\underline{\underline{5}}}\,dx$

$\square = 3x^2+9$

$\square' = 6x$

$\dfrac{1}{6} \times \dfrac{(3x^2+9)^6}{6}$

$\underline{\underline{Q}}$ $\int \sqrt{2x+1}\, dx$

$\frac{1}{2}\int 2\left(\boxed{2x+1}\right)^{\frac{1}{2}} dx$ $\qquad \square = 2x+1$

$\qquad \square' = 2$

$\frac{1}{2} \times \frac{(2x+1)^{1.5}}{1.5}$

$\frac{(2x+1)^{1.5}}{3}$

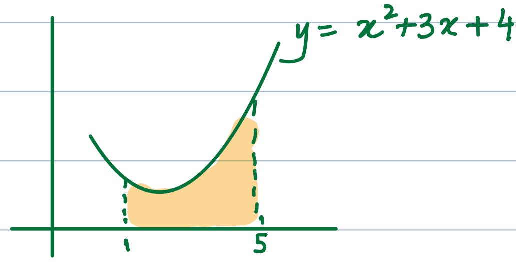
$\underline{\underline{Q}}$ $\int (2x^1+5)\, dx$

$\frac{2x^2}{2} + 5x$

$x^2 + 5x$

AREA UNDER GRAPH.

(Area between curve and x-axis)



Modulus

$|3| = 3$

$|-3| = 3$

$$\text{Area} = \int_1^5 (x^2+3x+4)\,dx$$

Modulus ←
$$\left| \frac{x^3}{3} + \frac{3x^2}{2} + 4x \right|_1^5$$

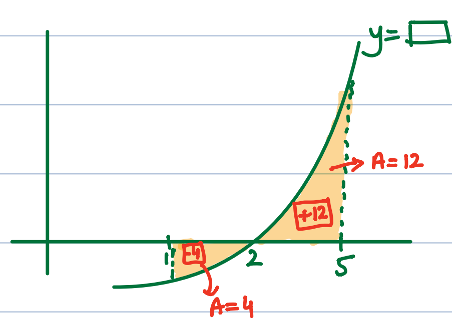
$$\left| \left(\frac{5^3}{3} + \frac{3(5)^2}{2} + 4(5) \right) - \left(\frac{1^3}{3} + \frac{3(1)^2}{2} + 4(1) \right) \right|$$

$$= |99.2 - 5.83|$$

$$\boxed{\text{Area} = 93.37}$$

VARIATION

1



$$\int_1^5 \square\,dx = 8 = +12 - 4 = 8$$
$$= |-4+12| = |8| = 8$$

For shaded area :-

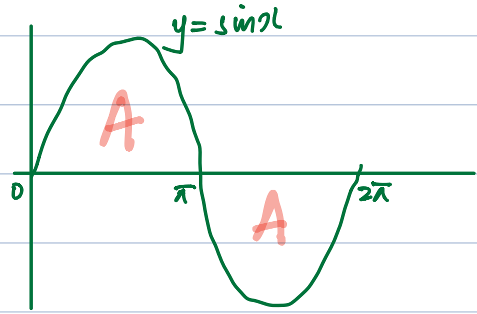
$$\int_1^2 \square\,dx + \int_2^5 \square\,dx$$

$$|-4| + |12|$$

$$= 4 + 12 = \boxed{16}$$

Integration considers areas below x-axis as negative. And it gives us net answer.

Hence if you have some portion of shaded area below x-axis, you will need to split the process.

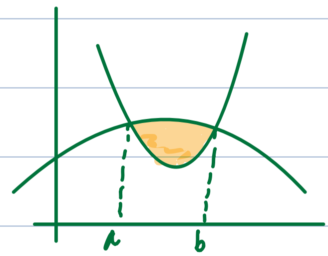


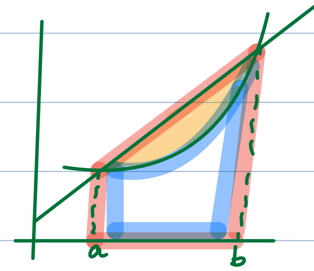
$$\int_0^{2\pi} \sin x \, dx = 0$$

$$\int_0^{\pi} \sin x \, dx = A$$

$$\int_{\pi}^{2\pi} \sin x \, dx = |-A| = A$$

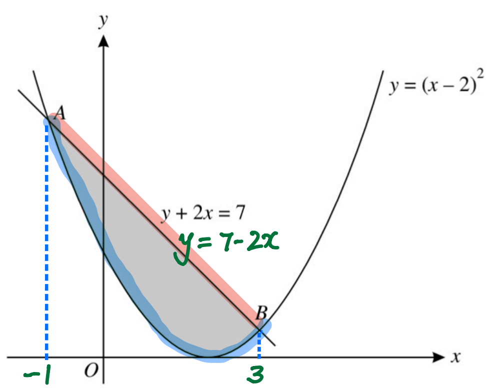
$\boxed{2}$ Area between two graphs





$$\text{Area between two graphs} = \int_a^b (\text{upper graph})\,dx - \int_a^b (\text{lower graph})\,dx.$$

21



The diagram shows the curve $y = (x - 2)^2$ and the line $y + 2x = 7$, which intersect at points A and B.
Find the area of the shaded region. [8]

Points of intersection:

$y = (x-2)^2 \qquad y = 7 - 2x$

$(x-2)^2 = 7 - 2x$

$x^2 - 4x + 4 = 7 - 2x$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x+1)(x-3)=0$

$x=-1 \quad , \quad x=3$

upper graph $\quad$ lower graph.

$$\text{Shaded Area} = \int_{-1}^{3} (7-2x)\,dx - \int_{-1}^{3} 1\left(\boxed{x-2}\right)^{2} dx$$

$\square = x-2$

$\square' = 1$

$$= \left| 7x - \frac{\cancel{2}x^2}{\cancel{2}} \right|_{-1}^{3} - \left| \frac{(x-2)^3}{3} \right|_{-1}^{3}$$

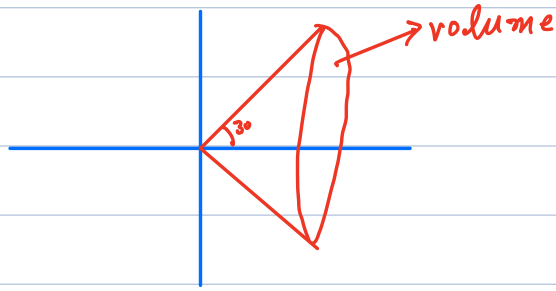
$$\left| \left(7(3)-(3)^2\right) - \left(7(-1)-(-1)^2\right) \right| - \left| \frac{(3-2)^3}{3} - \frac{(-1-2)^3}{3} \right|$$

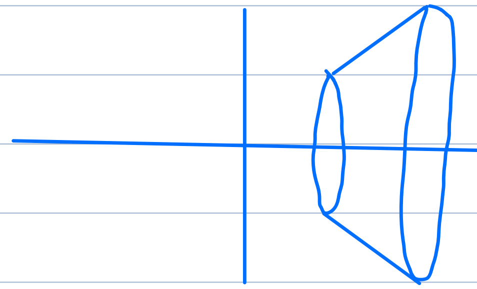
$$\left(12 - (-6)\right) - \left(\frac{1}{3} + 9\right)$$

$$= 18 - \frac{28}{3}$$

$$\text{Shaded area.} = \boxed{\frac{26}{3}}$$

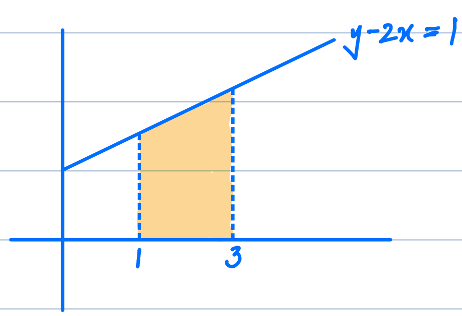
VOLUME OF ROTATION 360°





ABOUT x-AXIS:	ABOUT Y-AXIS :-
1) Make y-subject.	1) Make x-subject.
2) Square both sides.	2) Square both sides.
3) Integrate RHS	3) Integrate RHS
4) Apply limits	4) Apply limits
5) Multiply by π	5) Multiply by π

Q:



Find the volume when the shaded region is rotated 360° about x-axis.

Step1: Make y subject $\qquad y = 2x+1$

Step2: Square both sides $\qquad y^2 = (2x+1)^2$

Step3: Integrate RHS.

$$\frac{1}{2}\int 2\left(\boxed{2x+1}\right)^2 dx$$

$\square = 2x+1$

$\square' = 2$

$$\frac{1}{2} \times \frac{(2x+1)^3}{3}$$

Step4: Apply limits $\qquad \left| \frac{(2x+1)^3}{6} \right|_1^3$

$$\left| \frac{(2(3)+1)^3}{6} - \frac{(2(1)+1)^3}{6} \right|$$

$$\frac{316}{6}$$

$$52\frac{2}{3}$$

Step5: Multiply by π

$$\text{Volume} = 52\frac{2}{3} \times \pi = \boxed{165.45}$$

IN THIS WORKING SOMETIMES THE CONDITION FOR INTEGRATION WILL NOT BE GETTING SATISFIED. IN THAT CASE OPEN THE BRACKETS USING $(a \pm b)^2 = a^2 \pm 2ab + b^2$.

eg $\qquad \int (\boxed{2x - x^2})^2 \, dx$ $\qquad \square = 2x - x^2$

$\square' = 2 - 2x$

we cannot introduce a variable term.

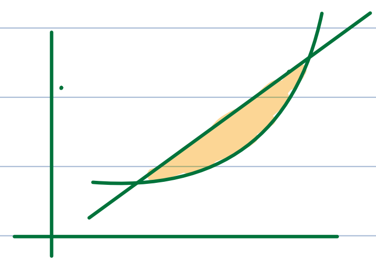
$$\int \left((2x)^2 - 2(2x)(x^2) + (x^2)^2 \right) dx$$

$$\int \left(4x^2 - 4x^3 + x^4 \right) dx$$

Now integrate individually.

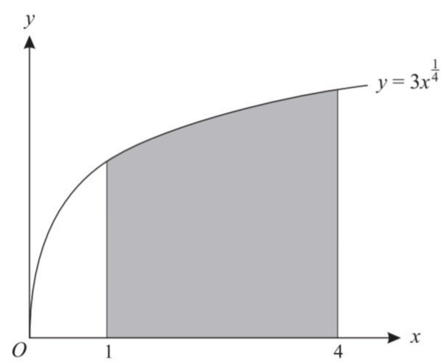
$$\frac{4x^3}{3} - \frac{4x^4}{4} + \frac{x^5}{5}$$

VOLUME OF SHADED REGION BETWEEN TWO GRAPHS:



Volume of shaded region = Volume of upper graph − Volume of lower graph.

12



The diagram shows the curve $y = 3x^{\frac{1}{4}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$. Find the volume of the solid obtained when this shaded region is rotated completely about the x-axis, giving your answer in terms of π. [4]

$$y = 3x^{\frac{1}{4}}$$

$$y^2 = \left(3x^{\frac{1}{4}}\right)^2$$

$$y^2 = 9x^{\frac{1}{2}}$$

$$\int 9x^{\frac{1}{2}}\,dx$$

$$\frac{9x^{1.5}}{1.5}$$

$$\left|6x^{1.5}\right|_1^4$$

$$\left|6(4)^{1.5} - 6(1)^{1.5}\right|$$

$$|48 - 6|$$

$$= 42$$

$$\times \pi$$

$$\text{Volume} = 42\pi$$

REVERSE STEP:

$$y \underset{\text{integrate}}{\overset{\text{diff}}{\rightleftarrows}} \frac{dy}{dx} \underset{\text{integrate}}{\overset{\text{diff}}{\rightleftarrows}} \frac{d^2y}{dx^2}$$

Constant of Integration (+c)

$$y = x^2 - 5x + 8$$

diff

$$\frac{dy}{dx} = 2x - 5$$

integrate

$$y = \int (2x-5)\,dx$$

$$y = \frac{2x^2}{2} - 5x$$

$$y = x^2 - 5x + c$$

NOTE :- 1) IF YOU ARE INTEGRATING "WITH LIMITS" (AREA OR VOLUME), NO NEED TO PUT +C

2) IF YOU ARE INTEGRATING "WITHOUT LIMITS" (REVERSE WORKING), ALWAYS USE +C

$$\left|\left(\quad \cancel{+c}\right) - \left(\quad +c\right)\right|$$

Q $\frac{dy}{dx} = 3x + 5$. Given that curve passes through $(1, 5)$, Find equation of curve.

$$y = \int (3x+5)\,dx$$

$$\boxed{y = \frac{3x^2}{2} + 5x + c}$$

$x = 1$
$y = 5$

Never use $y - y_1 = m(x - x_1)$ for these parts since that is equation of straight line. NOT of curve.

$$5 = \frac{3(1)^2}{2} + 5(1) + c$$

$$c = -\frac{3}{2}$$

EQUATION OF CURVE: $\boxed{y = \frac{3x^2}{2} + 5x - \frac{3}{2}}$

7 A curve is such that $\frac{dy}{dx} = \frac{16}{x^3}$, and $(1, 4)$ is a point on the curve.

$x \quad y$

(i) Find the equation of the curve. [4]

y

$y \xrightarrow{\text{diff}} \frac{dy}{dx}$, $\frac{dy}{dx} \xrightarrow{\text{integ.}} y$

$$y = \int \frac{16}{x^3}\,dx$$

$$y = \int 16x^{-3}\,dx$$

$$y = \frac{16x^{-2}}{-2} + c$$

$$\boxed{y = -\frac{8}{x^2} + c}$$

$x = 1$
$y = 4$

$$4 = -\frac{8}{(1)^2} + c$$

$$c = 12$$

CURVE

$$\boxed{y = -\frac{8}{x^2} + 12}$$

15 A curve is such that $\frac{dy}{dx} = 4 - x$ and the point $P(2, 9)$ lies on the curve. The normal to the curve at P meets the curve again at Q. Find

(i) the equation of the curve, [3]

$$y = \int (4 - x)\,dx$$

$$y = 4x - \frac{x^2}{2} + c$$

$$\boxed{y = 4x - \frac{x^2}{2} + c}$$

$x = 2$
$y = 9$

$$9 = 4(2) - \frac{2^2}{2} + c$$

$$9 = 8 - 2 + c$$

$$c = 3$$

$$\boxed{y = 4x - \frac{x^2}{2} + 3}$$

$$y \rightleftarrows \frac{dy}{dx} \rightleftarrows \frac{d^2y}{dx^2}$$

44 A curve is such that $\frac{d^2y}{dx^2} = -4x$. The curve has a maximum point at (2, 12).

$\frac{dy}{dx} = 0$, $x = 2$, $y = 12$

(i) Find the equation of the curve. [6]

y

$$\frac{dy}{dx} = \int(-4x)\,dx$$

$$\frac{dy}{dx} = -4\frac{x^2}{2} + c$$

$$\boxed{\frac{dy}{dx} = -2x^2 + c}$$

$\frac{dy}{dx} = 0$, $x = 2$

$$0 = -2(2)^2 + c$$

$$\boxed{c = 8}$$

$$\frac{dy}{dx} = -2x^2 + 8$$

$$y = \int(-2x^2 + 8)\,dx$$

$$\boxed{y = -\frac{2x^3}{3} + 8x + c}$$

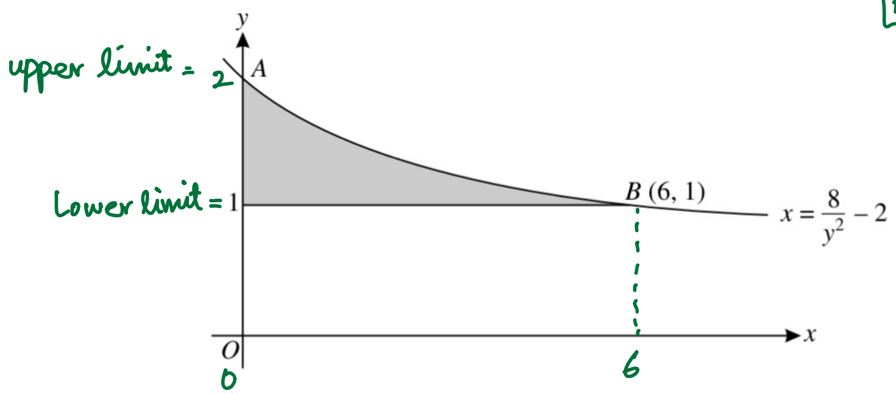
$x = 2$
$y = 12$

$$12 = -\frac{2(2)^3}{3} + 8(2) + c$$

$$c = \frac{16}{3} - 4 = \frac{4}{3}$$

$$\boxed{y = \frac{-2x^3}{3} + 8x + \frac{4}{3}}$$

43



$\boxed{A}$ $x = 0$

$0 = \frac{8}{y^2} - 2$

$2 = \frac{8}{y^2}$

$2y^2 = 8$

$y^2 = 4$

$y = 2$

The diagram shows part of the curve $x = \frac{8}{y^2} - 2$, crossing the y-axis at the point A. The point $B\,(6,\ 1)$ lies on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 1$. Find the exact volume obtained when this shaded region is rotated through 360° about the **y-axis**. [6]

$$x = \frac{8}{y^2} - 2$$

$$x^2 = \left(\frac{8}{y^2} - 2\right)^2$$

$$\int \left(\frac{8}{y^2} - 2\right)^2 dy$$

$$\left(\frac{8}{y^2}\right)^2 - 2\left(\frac{8}{y^2}\right)(2) + (2)^2$$

$$\frac{64}{y^4} - \frac{32}{y^2} + 4$$

$$\int \left(64y^{-4} - 32y^{-2} + 4\right) dy$$

$$\left| \frac{64y^{-3}}{-3} - \frac{32y^{-1}}{-1} + 4y \right|_1^2$$

$$\left|\left(\frac{64(2)^{-3}}{-3} - \frac{32(2)^{-1}}{-1} + 4(2)\right) - \left(\frac{64(1)^{-3}}{-3} - \frac{32(1)^{-1}}{-1} + 4(1)\right)\right|$$

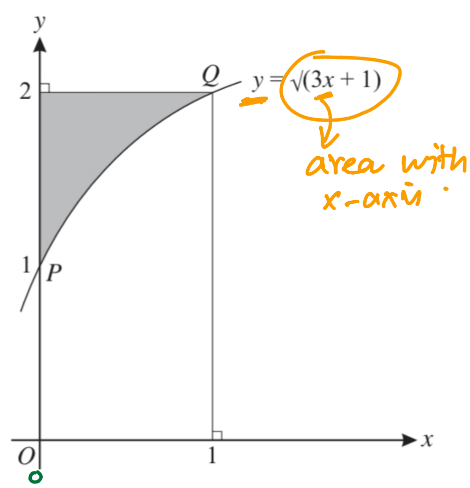
$$\left|\frac{64}{3} - \left(\frac{44}{3}\right)\right|$$

$$\frac{20}{3}$$

$$\times \pi$$

$$\text{Volume} = \frac{20\pi}{3}$$

17



The diagram shows the curve $y = \sqrt{(3x + 1)}$ and the points $P\,(0,\ 1)$ and $Q\,(1,\ 2)$ on the curve. The shaded region is bounded by the curve, the y-axis and the line $y = 2$.

(i) Find the area of the shaded region. [4]

Shaded area = Rectangle − Area under curve.

$$= (2)(1) - \frac{14}{9}$$

$$= 2 - \frac{14}{9} = \boxed{\frac{4}{9}}$$

Area under curve $= \frac{1}{3}\int 3\left(\boxed{3x+1}\right)^{\frac{1}{2}} dx$

$\square = 3x+1$

$\square' = 3$

$$\frac{1}{3} \times \frac{(3x+1)^{1.5}}{1.5}$$

$$\left| \frac{(3x+1)^{1.5}}{4.5} \right|_0^1$$

$$\left| \frac{(3(1)+1)^{1.5}}{4.5} - \frac{(3(0)+1)^{1.5}}{4.5} \right|$$

$$\frac{16}{9} - \frac{2}{9}$$

$$= \frac{14}{9}$$

Method 2: To Find area between curve and y-axis, make x- subject

$$y = \sqrt{3x+1}$$

$$y^2 = 3x+1$$

$$x = \frac{y^2 - 1}{3}$$

area with y-axis.

Integrate and put limits of y-axis now.

$$\int \frac{y^2-1}{3}\,dy$$

$$\frac{1}{3}\int (y^2-1)\,dy$$

$$\frac{1}{3}\left| \frac{y^3}{3} - y \right|_1^2$$

$$\frac{1}{3}\left|\left(\frac{2^3}{3}-2\right) - \left(\frac{1^3}{3}-1\right)\right|$$

$$\frac{1}{3}\left| \frac{2}{3} - \left(-\frac{2}{3}\right)\right|$$

$$\frac{1}{3}\left|\frac{4}{3}\right| = \boxed{\frac{4}{9}}$$

whenever there is a constant common in DIFF OR INTEG always take it outside.

Friday }
Saturday } Sequences

Sunday } Quadratics Marathon. (3hours).

Regular. evening Time is M1

Mon }
Tue } Functions Marathons (Topic + Practice).
Wed } (morning).